

# Tradable immigration quotas revisited

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## Abstract

This paper revisits the theory behind tradable immigration quotas, which have been proposed to raise South-North migration and, specifically, refugee resettlement. We qualify previous research by showing that tradable immigration quotas are generally not utilitarian efficient for host countries. The reason is that global migration entails more complex externalities than other public-good problems, such as environmental protection, where market-based instruments are common. The inefficiency is exacerbated when migrants are matched to countries. Despite these caveats, we offer new results in favor of tradable immigration quotas plus matching.

**Keywords:** immigration, asylum, tradable quotas, cap and trade, matching, externalities.

**JEL classification:** C78, D62, F22, F55, H23, H41.

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# 1 Introduction

High-income countries care about the living standards in less fortunate parts of the world and spend considerable sums of money to improve them. In 2020, for example, official development assistance from OECD members amounted to more than 160 billion US dollars (OECD, 2021). An alternative—and arguably more effective—way to help the global poor is migration (Clemens, 2011; Pritchett, 2006). Yet high-income countries severely limit the entry of low-skilled economic migrants as well as refugees. These restrictions reflect the perceived costs of immigration, including its labor-market impact and sociopolitical ramifications. But even from the perspective of destination countries, current immigration laws may be inefficiently tight because of the public-good nature of migration: since poor or persecuted people escape their plight if they receive shelter *somewhere* in the rich world, regardless of the specific place, each potential host country has an incentive to free-ride on its peers.

Fernández-Huertas Moraga and Rapoport (2014), henceforth FHM-R, make an innovative proposal to issue more visas to people in need. They suggest setting a target for South-North migration above the status-quo level and assigning each high-income country an initial quota of immigration slots; the sum of these quotas equals the target. In analogy to a cap-and-trade system for environmental regulation, countries can exchange their quotas in a competitive market. The resulting allocation determines how many immigrants each country receives. To fill the available slots, migrants are matched to countries according to both sides’ preferences over each other. Via a mathematical model, FHM-R argue that this mechanism is utilitarian efficient for rich countries, meaning that it maximizes their aggregate utility.

In multiple follow-up papers, Fernández-Huertas Moraga and Rapoport (2015a, 2015b, 2016) advocate reforming the EU asylum system via their mechanism. This application has raised interest among policy-makers in Europe, who invited the two economists to discuss their proposal.<sup>1</sup> While ethical objections were raised against a “market for refugees”, the European Parliament (2015) endorsed a rudimentary matching procedure to complement the relocation of asylum seekers within the EU. Migrants should rank countries and validate their preferences via family ties, language skills, pro-

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<sup>1</sup>Personal communication by Jesús Fernández-Huertas Moraga, 20 October 2021.

fessional qualifications, etc. This proposal eventually died in the Council of the European Union (2015) as national governments were reluctant to give more choice to asylum seekers. Neither matching nor tradable quotas feature in the European Commission’s (2020) latest reform package, the New Pact on Migration and Asylum, but it puts forward two related ideas. First, asylum seekers should be hosted by countries with which they have “meaningful links” that could foster their integration prospects. Second, EU member states should be allowed to choose between relocation and other forms of “solidarity”, such as the deportation of rejected asylum seekers.

To inform the political debate, it is crucial to understand the theoretical merits of tradable immigration quotas and matching. In this paper, we revisit FHM-R’s analysis and show that the model predictions are more nuanced than their results suggest. Our main finding is that the logic why cap-and-trade systems efficiently curb pollution at the national level does not extend to global migration policy. The cross-border externalities are generally too complex to be addressed by market mechanisms.

Cap-and-trade systems are primarily used to regulate pollution by firms within a given country (Tietenberg, 2010). The usual assumption in economic models is that firms emitting, say, greenhouse gases harm society at large but do not care themselves about pollution. Utilitarian efficiency requires firms to cut emissions until their marginal costs of abatement equal society’s marginal benefit. In the absence of regulation, firms pollute too much because they disregard their environmental impact. A cap-and-trade system can achieve efficiency by limiting total emissions and putting a price on pollution or, equivalently, a reward on abatement. Firms for which it is relatively cheap to reduce emissions will optimally sell allowances to firms that struggle to pollute less. In equilibrium, each firm’s marginal cost of abatement equals the market price. If the cap on total permits is set correctly, the market price reflects society’s marginal benefit of emission cuts, thus yielding efficiency.

This conclusion does not carry over to a cap-and-trade system for global migration.<sup>2</sup> The key difference is that, in FHM-R’s model, countries impose externalities *on each other*: they all gain when more destitute people are allowed to move to the rich world. Each country’s individually optimal action

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<sup>2</sup>The “cap” in FHM-R’s proposal is actually a *lower*, not an upper, bound because immigration generates a *positive*, not a negative, externality.

balances its own benefit and cost of immigration but disregards the other countries' benefits. In the market equilibrium, countries' marginal costs net of their marginal *private* benefits coincide. By contrast, efficiency requires equalization of countries' marginal costs net of the marginal *social* benefit. In the generic case where the marginal private benefits of immigration vary across countries, the market equilibrium will fail to be efficient ([Proposition 1](#)). This issue does not occur in the standard model of environmental regulation because firms obtain no direct benefit from emission cuts.

Nevertheless, we offer several results in defense of tradable immigration quotas. The first one is that the inefficiency of the market equilibrium may be seen as an oddity of Nash behavior. [Proposition 1](#) assumes, as is standard, that each country best responds to the other countries' actions (i.e. their immigration levels). Implicitly, thus, countries believe that their market decisions can change the total number of immigrants. They should know, however, that a cap-and-trade system fixes the quantity of available visas. Motivated by this incongruity, we propose an alternative notion of market equilibrium in which it is the total level of immigration that countries take as given. Under this solution concept, each country knows that accepting more immigrants itself will reduce the other countries' intake by the same amount. The benefits, which we assume to depend on the total level of immigration, are therefore unaffected by the market. Each country optimally sets its marginal cost equal to the market price, just like firms in a cap-and-trade system for pollution control. Hence, the market equilibrium will be efficient under a plausible form of non-Nash behavior ([Proposition 2](#)).

Sticking to Nash equilibrium, efficiency can still be achieved—via a market with personalized prices. In analogy to “trading ratios” in environmental economics (Farrow et al., [2005](#); Holland and Yates, [2015](#); Montgomery, [1972](#); Muller and Mendelsohn, [2009](#)), the idea is to set individual exchange rates between quotas and immigration levels. The less a country benefits from immigration, the more quotas it must trade away—and thus the more it pays—to reduce its intake of migrants by a given amount. This tweak harmonizes incentives across countries, ensuring an efficient market equilibrium exists ([Proposition 3](#)). To set the exchange rates correctly, however, the planner must know countries' preferences, which is arguably unrealistic.

Although a standard market for immigration quotas is generally not efficient, we prove that it raises countries' aggregate utility above the status-quo

level if we make the common assumption that the benefit functions are linear and the cost functions are quadratic (Proposition 4). Moreover, by adjusting the initial quotas, the overall welfare gain can be distributed so that all countries prefer the market equilibrium to the status quo (Proposition 5). Since participation in international agreements is voluntary, it matters that the reform proposal can make all countries better off. That said, the informational caveat from above applies here as well: the planner must know the shape of countries' benefit and cost functions to find the right initial quotas.

Further complications arise when tradable immigration quotas are combined with a matching mechanism. We mostly focus on one-sided matching in which migrants prefer some destinations over others, while countries care only about how many people they receive. The main problem is that an attractive country which extends its immigration capacity may woo people away from a less popular destination. If the latter cannot find enough other migrants to fill the open slots, its utility will decrease. These "matching externalities" are not internalized by individual countries, which creates another source of inefficiency. Indeed, we show that tradable immigration quotas may fail to be efficient subject to the matching constraints, even if efficiency would prevail in the absence of matching (Example 4). That said, if we assume that countries are always able to attract their desired number of migrants, then matching does not affect the market's welfare properties (Proposition 6). This scenario does not seem unrealistic, given that more than 80 million people are forcibly displaced (UNHCR, 2021) and 750 million people would migrate if they could (Esipova et al., 2018).

In sum, the application of cap and trade, plus matching, to global migration policy is more complex than previously thought. The issues that we identify do not afflict the recent literature on refugee matching where countries have *fixed* capacities for immigrants (Andersson and Ehlers, 2020; Andersson et al., 2018; Aziz et al., 2018; Delacrétaz et al., 2020; Jones and Teytelboym, 2016, 2017a, 2017b, 2018; van Basshuysen, 2017).<sup>3</sup> Assuming fixed capacities is reasonable when refugees are allocated within a given country because national governments can often impose quotas on subordinate entities, such as regions or cities. At the international level, how-

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<sup>3</sup>See Andersson (2019) for a brief survey. A related, more data-driven approach is to algorithmically assign future refugee cohorts on the basis of past integration outcomes (Ahani et al., 2021; Bansak et al., 2018).

ever, most of the political debate about asylum policy has centered on *how many*—rather than *which*—refugees each country should take (Barigazzi & de la Baume, 2015). The quantitative dimension that FHM-R introduce, and we enlarge on, is therefore practically relevant.

## 2 Basic theory

We now introduce the baseline model and study the conditions under which tradable immigration quotas are efficient.

### 2.1 Model

There are  $n \in \{2, 3, \dots\}$  host countries and a pool of potential migrants with infinite Lebesgue measure. The infinity assumption is technically convenient and captures the idea that more people are willing to migrate than the  $n$  countries can possibly host. For now, all migrants find all countries acceptable. This assumption will be relaxed in Section 4. Similarly, each country  $i \in \{1, \dots, n\}$  cares only about the number of immigrants, not their composition. Its preferences are characterized by a cost function  $c_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  and a benefit function  $b_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ , where  $\mathbb{R}_+ := [0, \infty)$ .

When country  $i$  hosts  $m_i \geq 0$  migrants, it incurs a cost  $c_i(m_i) \geq 0$ . This cost reflects all factors that shape the government’s perception of immigration: the financial expenses for housing, health care or language courses; the fiscal and labor-market impact; but also the sociopolitical ramifications, such as the rise of anti-immigration parties or the erosion of cultural homogeneity. Low-skilled and forced migrants are often perceived as a burden, which explains why the cost is non-negative. We further make the following assumptions: First,  $c_i(0) = 0$ , that is,  $i$  incurs no cost if it hosts zero immigrants. Second,  $c_i$  is strictly increasing, so more immigrants imply higher costs. Third,  $c_i$  is strictly convex, meaning that  $i$ ’s marginal cost of receiving one additional person grows with its intake of immigrants. For technical reasons, we also require  $c_i$  to be continuously differentiable,  $c_i'(0) = 0$  and  $\lim_{m_i \rightarrow \infty} c_i'(m_i) = \infty$ .

Immigration is not only costly but also beneficial to governments that care about the plight of the global poor. These gains accrue to a country both when it provides visas itself and when other countries do so. Formally, the benefit  $b_i(\sum_j m_j) \geq 0$  that country  $i$  derives from immigration depends

on the global level  $\sum_j m_j$ , not just  $i$ 's own contribution  $m_i$ . Accordingly, immigration is a public good. We assume that each  $b_i$  is strictly increasing, weakly concave and continuously differentiable. Hence, the marginal benefit is positive and weakly decreases as more and more migrants receive visas.

In a sense, we are considering a “pure” public good because the benefits are determined by the total number of immigrants, not their distribution across countries. This assumption is the main difference to FHM-R’s model, which allows country  $i$  to benefit more—or less—from its own contribution  $m_i$  than the other countries’ joint contribution  $\sum_{j \neq i} m_j$ . FHM-R further assume  $i$ 's benefit function to be additively separable in  $m_i$  and  $\sum_{j \neq i} m_j$ , which means that  $i$ 's marginal benefit from hosting more people itself does not depend on the other countries’ immigration levels. As we discuss in [Section 3.5](#), our results carry over to FHM-R’s model.

Occasionally, we will use the term “allocation” to refer to a vector of migrants  $\mathbf{m} := (m_i)_{i=1}^n \in \mathbb{R}_+^n$ . Countries’ preferences are assumed to be quasilinear in money. That is, if the allocation is  $\mathbf{m}$  and country  $i$  receives a monetary transfer  $t_i \in \mathbb{R}$ , then  $i$ 's utility is  $b_i(\sum_j m_j) - c_i(m_i) + t_i$ . All of the above is common knowledge.

## 2.2 Inefficiency of the status quo

We start the analysis by characterizing the status quo, which is the pure-strategy Nash equilibrium of the game in which every country chooses its level of immigration non-cooperatively.

**Definition 1.** An allocation  $\mathbf{m}^s := (m_i^s)_{i=1}^n \in \mathbb{R}_+^n$  is a *status quo* if

$$\forall i, \quad m_i^s \in \arg \max_{m_i \geq 0} \left\{ b_i \left( m_i + \sum_{j \neq i} m_j^s \right) - c_i(m_i) \right\}. \quad (1)$$

**Lemma 1.** *There exists a unique status quo  $\mathbf{m}^s$ . Defining  $M^s := \sum_i m_i^s$ ,  $\mathbf{m}^s$  is characterized by*

$$\forall i, \quad c'_i(m_i^s) = b'_i(M^s). \quad (2)$$

The proof is in [Appendix A.1](#). Equation (2) says that, in the status quo, each country makes its marginal cost equal to its marginal benefit. It thus ignores that raising its level of immigration is beneficial for the other countries as well. For this reason, the status quo fails to be efficient in the utilitarian

sense, meaning that it does not maximize the countries' aggregate utility. Specifically, total immigration in the status quo is below the efficient level.

**Definition 2.** An allocation  $\mathbf{m}^e := (m_i^e)_{i=1}^n \in \mathbb{R}_+^n$  is (utilitarian) *efficient*, or a (utilitarian) *optimum*, if

$$\mathbf{m}^e \in \arg \max_{\mathbf{m} \in \mathbb{R}_+^n} \sum_i \left[ b_i \left( \sum_j m_j \right) - c_i(m_i) \right]. \quad (3)$$

**Lemma 2.** *There exists a unique optimum  $\mathbf{m}^e$ . Defining  $M^e := \sum_i m_i^e$ ,  $\mathbf{m}^e$  is characterized by*

$$\forall i, \quad c'_i(m_i^e) = \sum_j b'_j(M^e). \quad (4)$$

Moreover,  $M^e > M^s$ .

We prove this result in [Appendix A.2](#). The difference between (4) and (2) is that efficiency requires each country to equate its marginal cost to the marginal *social* benefit rather than to its marginal *private* benefit. Note that the right-hand side of (4) is the same for all countries, so the optimum equalizes their marginal costs.

By [Lemma 2](#), countries collectively gain when total immigration is raised from  $M^s$  to  $M^e$ . Following FHM-R, we allow for the possibility that—for exogenous reasons—total immigration should not exceed some threshold  $M \geq M^s$ . An allocation that maximizes utilitarian welfare subject to this constraint is said to be constrained efficient.

**Definition 3.** Given  $M \geq M^s$ , an allocation  $\mathbf{m}^c(M) := (m_i^c(M))_{i=1}^n \in \mathbb{R}_+^n$  is *constrained efficient*, or a *constrained optimum*, if

$$\mathbf{m}^c(M) \in \arg \max_{\mathbf{m} \in \mathbb{R}_+^n} \sum_i \left[ b_i \left( \sum_j m_j \right) - c_i(m_i) \right] \quad \text{s.t.} \quad \sum_i m_i \leq M. \quad (5)$$

Since  $\sum_i m_i^s = M^s \leq M$ ,  $\mathbf{m}^s$  is in the feasible set of this maximization problem. Hence, a constrained optimum yields at least as much aggregate utility as the status quo. Our next result characterizes the solution to (5), which exists and is unique. If  $M \geq M^e$ , then the constraint  $\sum_i m_i \leq M$  is not binding, so  $\mathbf{m}^c(M) = \mathbf{m}^e$ . While efficiency is unattainable for  $M < M^e$ , the constrained optimum still equalizes marginal costs across countries.

**Lemma 3.** For each  $M \geq M^s$ , there exists a unique constrained optimum  $\mathbf{m}^c(M)$ . It is characterized by  $\sum_i m_i^c(M) = \min\{M, M^e\}$  and

$$\forall i, \forall j, \quad c'_i(m_i^c(M)) = c'_j(m_j^c(M)). \quad (6)$$

For the proof, see [Appendix A.3](#).

### 2.3 Tradable immigration quotas

To ameliorate the inefficiency of the status quo, FHM-R propose creating tradable immigration quotas. They work as follows: First, the desired level of total immigration  $M \geq M^s$  is set. Each country  $i$  gets an initial quota of immigrants  $m_i^0(M) \geq 0$  such that  $\sum_i m_i^0(M) = M$ . These quotas can be traded in a competitive market with price  $p(M) \geq 0$ .

The standard notion of competitive equilibrium in markets with externalities essentially marries Walras and Nash: when contemplating its optimal demand for quotas, each country considers the price and the other countries' actions as given. In equilibrium, countries' aggregate demand must coincide with the available number of quotas  $M$  (cf. Arrow and Hahn, 1971, ch. 6.2; Laffont, 1988, ch. 1.3). An alternative equilibrium definition, with different welfare implications, will be presented in [Section 3.1](#).

**Definition 4.** Given  $M \geq M^s$  and  $(m_i^0(M))_{i=1}^n \in \mathbb{R}_+^n$  such that  $\sum_i m_i^0(M) = M$ , a *market equilibrium*  $(\mathbf{m}^m(M); p^m(M))$  consists of an allocation  $\mathbf{m}^m(M) := (m_i^m(M))_{i=1}^n \in \mathbb{R}_+^n$  and a price  $p^m(M) \geq 0$  such that

$$\forall i, \quad m_i^m(M) \in \arg \max_{m_i \geq 0} \left\{ b_i \left( m_i + \sum_{j \neq i} m_j^m(M) \right) - c_i(m_i) + p^m(M) [m_i - m_i^0(M)] \right\} \quad (7)$$

and  $\sum_i m_i^m(M) = M$ .

**Lemma 4.** For each  $M \geq M^s$ , there exists a unique market equilibrium  $(\mathbf{m}^m(M); p^m(M))$ . It satisfies

$$\forall i, \quad c'_i(m_i^m(M)) = b'_i(M) + p^m(M). \quad (8)$$

[Appendix A.4](#) contains the proof. Comparing [Lemmas 3](#) and [4](#), we see that the constrained optimum equalizes marginal costs across countries, whereas

the market equilibrium may not. Thus, in general,  $\mathbf{m}^m(M) \neq \mathbf{m}^c(M)$ . To better understand why, let us consider a numerical example.

**Example 1.** There are two countries ( $n = 2$ ) with identical cost functions but different benefit functions. Specifically, suppose that, for both  $i$  and all  $m_i, m_1, m_2 \geq 0$ ,  $c_i(m_i) = \frac{1}{2}(m_i)^2$  and  $b_i(m_1 + m_2) = \beta_i \cdot (m_1 + m_2)$ , where  $\beta_i > 0$  is a parameter measuring  $i$ 's marginal benefit from total immigration.

Applying (2) and (4),  $m_i^s = \beta_i$  and  $m_i^e = \beta_1 + \beta_2$  for both  $i$ . Thus,  $M^s = \beta_1 + \beta_2$  and  $M^e = 2(\beta_1 + \beta_2)$ . Suppose we wish to raise total immigration from the status-quo level  $M^s$  to some  $M \in [M^s, M^e]$ . By (6), the constrained optimum equalizes the two countries' marginal costs. Since their cost functions are identical, it follows that

$$m_1^c(M) = m_2^c(M) = M/2.$$

In contrast, (8) implies that the market equilibrium equalizes the differences between marginal cost and marginal benefit, that is,  $c'_1(m_1^m(M)) - b'_1(M) = c'_2(m_2^m(M)) - b'_2(M)$ . Given the functional forms, this equation becomes  $m_1^m(M) - \beta_1 = m_2^m(M) - \beta_2$ . By market clearing,  $m_1^m(M) + m_2^m(M) = M$ . Taken together, we get that

$$m_1^m(M) = \frac{M + \beta_1 - \beta_2}{2}, \quad m_2^m(M) = \frac{M + \beta_2 - \beta_1}{2}.$$

Note that  $m_1^m(M) = m_2^m(M)$  if and only if  $\beta_1 = \beta_2$ . Hence, the market equilibrium is constrained efficient if and only if both countries value immigration equally. Whenever country 1's marginal benefit exceeds country 2's (i.e.  $\beta_1 > \beta_2$ ), country 1 will receive more immigrants in the market equilibrium than country 2 (i.e.  $m_1^m(M) > M/2 > m_2^m(M)$ ), and vice versa.  $\triangle$

The inefficiency of the market equilibrium, illustrated in [Example 1](#), reflects the textbook insight that markets “fail” in the presence of externalities (Mas-Colell et al., 1995, ch. 11). Each country acts in its own interest, setting its marginal private disutility from immigration,  $c'_i(m_i^m(M)) - b'_i(M)$ , equal to the market price. The social planner, by contrast, also considers the benefits that the other countries reap from  $i$ 's contribution and thus equalizes countries' marginal *social* disutilities,  $c'_i(m_i^c(M)) - \sum_j b'_j(M)$ , which amounts to equalizing their marginal costs. Misalignment of the private and social incentives implies that the market equilibrium is not constrained efficient.

The sole exception is when all countries equally benefit from immigration.

**Proposition 1.** *For all  $M \in [M^s, M^e]$ ,  $\mathbf{m}^m(M) = \mathbf{m}^c(M)$  if and only if  $b'_i(M) = b'_j(M)$  for all  $i, j$ .*

*Proof.* Consider any  $M \in [M^s, M^e]$ . First, suppose that  $b'_i(M) = b'_j(M)$  for all  $i, j$ . Then, by (8),  $c'_i(m_i^m(M)) = c'_j(m_j^m(M))$  for all  $i, j$ . Market clearing requires that  $\sum_i m_i^m(M) = M$ . These are precisely the conditions that characterize the constrained optimum; see Lemma 3. Thus,  $\mathbf{m}^m(M) = \mathbf{m}^c(M)$ . Conversely, suppose that  $b'_i(M) \neq b'_j(M)$  for some  $i, j$ . From (8), it follows that  $c'_i(m_i^m(M)) \neq c'_j(m_j^m(M))$ . However, by (6),  $c'_i(m_i^c(M)) = c'_j(m_j^c(M))$ . Hence,  $\mathbf{m}^m(M) \neq \mathbf{m}^c(M)$ .  $\square$

Why, then, are markets often touted as efficient means to curb pollution and other environmental harms? The difference with global migration policy lies in the nature of the externalities. Cap and trade is primarily applied to firms within a given country (Tietenberg, 2010).<sup>4</sup> Models of domestic environmental regulation usually assume that polluting firms exert a negative externality on society, not on other firms (Stranlund, 2019, ch. 2). Hence, those who cause the externality are not those who suffer it. Translated to our model, this assumption would mean that national governments do not value immigration, so  $b_i = 0$  for all  $i$ , but the world population as a whole gets some benefit  $b(M) \geq 0$  when a total of  $M$  persons receive visas. The first-order conditions of the market equilibrium would then read  $c'_i(m_i^m(M)) = p^m(M)$  for all  $i$ . Hence, the market equilibrium would equalize marginal costs, just like the constrained optimum.

### 3 More on tradable immigration quotas

This section offers new perspectives on tradable immigration quotas, moving beyond the negative result of Proposition 1.

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<sup>4</sup>Cap and trade at the international level has so far been limited to the EU's Emissions Trading System, in which European *firms* buy and sell permits. Emissions trading between *governments*, as envisaged by Article 17 of the Kyoto Protocol, never occurred in its pure form. However, some countries fulfilled part of their Kyoto commitments by paying others to carry out "Green Investments Schemes" (Carbon Trust, 2009, ch. 4).

### 3.1 Alternative notion of market equilibrium

We start by proposing an alternative notion of market equilibrium that is constrained efficient. As the following example illustrates, the standard concept of competitive equilibrium with externalities (Definition 4) has an unappealing feature: it does not necessarily yield a Pareto improvement upon the initial quotas.

**Example 2.** There are two countries with identical cost and benefit functions:  $c_i(m_i) = \frac{1}{2}(m_i)^2$  and  $b_i(m_1 + m_2) = m_1 + m_2$  for both  $i$  and all  $m_i, m_1, m_2 \geq 0$ . It is easy to see that  $m_1^e = m_2^e = 2$ , so  $M^e = 4$ . The market equilibrium with  $M = M^e$  is given by  $m_1^m(M^e) = m_2^m(M^e) = 2$  and  $p^m(M^e) = 1$ . For expositional ease, let us suppress notation of  $M^e$ . Country  $i$  weakly prefers the market equilibrium to its initial quota  $m_i^0$  if and only if

$$b_i(m_1^m + m_2^m) - c_i(m_i^m) + p^m \cdot (m_i^m - m_i^0) \geq b_i(m_1^0 + m_2^0) - c_i(m_i^0).$$

Since  $m_1^m + m_2^m = m_1^0 + m_2^0 = 4$ , this inequality simplifies to

$$p^m \cdot (m_i^m - m_i^0) \geq c_i(m_i^m) - c_i(m_i^0) \iff m_i^0 \notin (0, 2).$$

Hence, unless  $\mathbf{m}^0 \in \{(0, 4), (2, 2), (4, 0)\}$ , one of the two countries loses from trading the initial quotas.  $\triangle$

Example 2 appears counter-intuitive: since trade is voluntary, how can some countries end up worse off than if they had kept their initial quotas? The explanation is that, by Nash reasoning, each country  $i$  takes  $\sum_{j \neq i} m_j$  as given and thus assumes that a raise in  $m_i$  will increase  $\sum_j m_j$  by the same amount. However, countries should understand that the total level of immigration is fixed at  $M$  because the market merely redistributes the initial quotas. Whatever decision  $i$  takes in the market, its benefit from immigration will always be  $b_i(M)$ . This logic suggests an equilibrium concept where countries take the total level of immigration, rather than that of the other countries, as given. That is, when pondering its optimal demand for quotas, country  $i$  assumes that an increase in  $m_i$  will lead to an equal decrease in  $\sum_{j \neq i} m_j$ , so that  $\sum_j m_j = M$  still holds.

**Definition 5.** Given  $M \geq M^s$  and  $(m_i^0(M))_{i=1}^n \in \mathbb{R}_+^n$  such that  $\sum_i m_i^0(M) = M$ , an *alternative market equilibrium*  $(\mathbf{m}^a(M); p^a(M))$  consists of an allo-

cation  $\mathbf{m}^a(M) := (m_i^a(M))_{i=1}^n \in \mathbb{R}_+^n$  and a price  $p^a(M) \geq 0$  such that

$$\forall i, \quad m_i^a(M) \in \arg \max_{m_i \geq 0} \left\{ b_i(M) - c_i(m_i) + p^a(M) [m_i - m_i^0(M)] \right\} \quad (9)$$

and  $\sum_i m_i^a(M) = M$ .

It follows immediately from (9) that, at price  $p^a(M)$ ,  $\mathbf{m}^a(M)$  yields at least as much utility for each country as  $\mathbf{m}^0(M)$ . In other words, the alternative market equilibrium is individually rational with respect to the initial quotas. Moreover, each country optimally sets its marginal cost equal to the price.

**Lemma 5.** *For each  $M \geq M^s$ , there exists a unique alternative market equilibrium  $(\mathbf{m}^a(M); p^a(M))$ . It satisfies*

$$\forall i, \quad c'_i(m_i^a(M)) = p^a(M). \quad (10)$$

The proof is in [Appendix A.5](#). Comparing [Lemmas 3](#) and [5](#), we see that the alternative market equilibrium is constrained efficient because it equalizes marginal costs across countries.

**Proposition 2.** *For all  $M \in [M^s, M^e]$ ,  $\mathbf{m}^a(M) = \mathbf{m}^c(M)$ .*

*Proof.* Consider any  $M \in [M^s, M^e]$ . [Definition 5](#) and [Lemma 5](#) imply that  $\mathbf{m}^a(M)$  is pinned down by  $\sum_i m_i^a(M) = M$  and  $c'_i(m_i^a(M)) = c'_j(m_j^a(M))$  for all  $i, j$ . The same conditions characterize  $\mathbf{m}^c(M)$ ; see [Lemma 3](#). Hence,  $\mathbf{m}^a(M) = \mathbf{m}^c(M)$ .  $\square$

By contrast, the standard market equilibrium is generally not constrained efficient ([Proposition 1](#)). These fundamentally different predictions raise the question which of the two equilibrium notions, if any, adequately represents real-life behavior in pure exchange economies with externalities. Empirical research can hopefully shed light on this issue.

### 3.2 Trading ratios

We now return to the paradigm of Nash behavior and, inspired by the idea of “trading ratios” from environmental economics, present a more sophisticated market design that achieves constrained efficiency.

Let  $M \in [M^s, M^e]$  be the desired level of total immigration. Define

$$\forall i, \quad r_i(M) := c'_i(m_i^c(M)) - b'_i(M), \quad (11)$$

which is country  $i$ 's marginal disutility at the constrained optimum.<sup>5</sup> The idea is to create  $Q(M) \in \mathbb{R}$  units of a virtual commodity. Specifically,

$$Q(M) := \sum_i r_i(M) m_i^c(M). \quad (12)$$

Each country  $i$  is assigned an initial quota  $q_i^0(M) \in \mathbb{R}$  such that  $\sum_i q_i^0(M) = Q(M)$ . These quotas can be traded in a competitive market with price  $p^r(M) \geq 0$ . Holding a quota  $q_i \in \mathbb{R}$  obliges  $i$  to host a certain number of immigrants  $m_i \geq 0$ , according to  $q_i = r_i(M) m_i$ . To reduce its level of immigration by one,  $i$  must give  $r_i(M)$  quotas to some other country  $j$ , along with a payment  $p^r(M) r_i(M)$ . Since  $j$  now holds  $r_i(M)$  additional quotas, it must host  $r_i(M)/r_j(M)$  more immigrants. This quotient is called the “trading ratio” in the environmental literature (e.g. Farrow et al., 2005).

Recall from Lemma 3 that  $c'_i(m_i^c(M)) = c'_j(m_j^c(M))$ . Hence, by (11),  $r_i(M)/r_j(M) = 1$  if and only if  $b'_i(M) = b'_j(M)$ . In this case, countries  $i$  and  $j$  trade visas on a one-for-one basis, just like in a standard market. But if their marginal benefits differ, then so do their exchange rates between quotas and visas (i.e.  $r_i$ ). While the price per quota (i.e.  $p^r$ ) is the same for all countries, the prices per visa (i.e.  $p^r r_i$ ) may vary. For example, suppose that  $i$  benefits less from immigration than  $j$ , that is,  $b'_i(M) < b'_j(M)$ . Then  $r_i(M) > r_j(M)$ , so  $i$  is paid more per visa than  $j$ . This tweak induces  $i$  to raise its immigration compared to a standard market, resulting in constrained efficiency.

**Definition 6.** Consider any  $M \geq M^s$ . Define  $(r_i(M))_{i=1}^n \in \mathbb{R}^n$  by (11), and  $Q(M) \in \mathbb{R}$  by (12). Let  $(q_i^0(M))_{i=1}^n \in \mathbb{R}^n$  be such that  $\sum_i q_i^0(M) = Q(M)$ . A *market equilibrium with trading ratios*  $(\mathbf{m}^r(M); p^r(M))$  consists of an allocation  $\mathbf{m}^r(M) := (m_i^r(M))_{i=1}^n \in \mathbb{R}_+^n$  and a price  $p^r(M) \geq 0$  such that

$$\forall i, \quad m_i^r(M) \in \arg \max_{m_i \geq 0} \left\{ b_i \left( m_i + \sum_{j \neq i} m_j^r(M) \right) - c_i(m_i) + p^r(M) [r_i(M) m_i - q_i^0(M)] \right\} \quad (13)$$

<sup>5</sup>Note that the right-hand side of (11) may be negative. However, if  $m_i^c(M) \geq m_i^s$ , then  $r_i(M) = c'_i(m_i^c(M)) - b'_i(M) \geq c'_i(m_i^s) - b'_i(M^s) = 0$ .

and  $\sum_i r_i(M)m_i^r(M) = Q(M)$ .

**Proposition 3.** *For all  $M \in [M^s, M^e]$ ,  $(\mathbf{m}^c(M); 1)$  is a market equilibrium with trading ratios.*

*Proof.* Consider any  $M \in [M^s, M^e]$  and let us suppress its notation. The first-order conditions of (13), which are both necessary and sufficient, read

$$\forall i, \quad b'_i\left(\sum_j m_j^r\right) - c'_i(m_i^r) + p^r r_i \leq 0, \quad (14)$$

with equality if  $m_i^r > 0$ . Set  $\mathbf{m}^r := \mathbf{m}^c$  and  $p^r := 1$ . Since  $\sum_j m_j^c = M$  and  $r_i = c'_i(m_i^c) - b'_i(M)$ , (14) holds with equality for all  $i$ . Moreover,  $\sum_i r_i m_i^r = \sum_i r_i m_i^c = Q$ , so the quota market clears. Therefore,  $(\mathbf{m}^c; 1)$  constitutes a market equilibrium with trading ratios.  $\square$

In environmental economics, markets with trading ratios have been proposed to regulate “non-uniformly mixed pollutants”, such as sulfur dioxide or nitrogen oxide. Unlike greenhouse gases, their damage is locally concentrated: emissions that occur closer to the monitoring points affect ambient quality more. The corresponding assumption in our model would be that, while  $b_i = 0$  for all  $i$ , the world population obtains a benefit  $b(\mathbf{m}) \geq 0$  that depends on the distribution of immigrants across countries. In this case, a standard cap-and-trade system does not ensure efficiency because countries/firms may create different marginal social benefits  $\partial b(\mathbf{m})/\partial m_i$ . By means of trading ratios or personalized prices, however, non-uniformly mixed pollutants can be regulated efficiently (Montgomery, 1972; Muller and Mendelsohn, 2009). The approach by Holland and Yates (2015) is particularly close to ours above.

While Proposition 3 is a nice theoretical result, its practical use is limited by the underlying informational assumptions. To set the total number of quotas and the exchange rates correctly, the planner must know the precise shape of countries’ cost and benefit functions—which seems unrealistic.

### 3.3 Improvement over the status quo

We know from Proposition 1 that, when countries are Nash optimizers and trading ratios are unfeasible, a market for visas will generally not be constrained efficient. Nonetheless, under the common assumption that the ben-

efit functions are linear and the cost functions are quadratic, tradable immigration quotas raise countries' aggregate utility above the status-quo level.<sup>6</sup>

**Proposition 4.** *Suppose that, for all  $i$ , there exist  $\beta_i > 0$  and  $\gamma_i > 0$  such that  $b_i(\sum_j m_j) = \beta_i \sum_j m_j$  and  $c_i(m_i) = \frac{1}{2}\gamma_i(m_i)^2$  for all  $\mathbf{m} \in \mathbb{R}_+^n$ . Then there exists  $\bar{M} > M^e$  such that for all  $M \in (M^s, \bar{M})$ ,*

$$\sum_i [b_i(M) - c_i(m_i^m(M))] > \sum_i [b_i(M^s) - c_i(m_i^s)].$$

The proof is in [Appendix A.6](#). To grasp the intuition, suppose we allocate  $M \geq M^s$  migrants via the market and then infinitesimally raise  $M$ . Countries' aggregate utility changes by

$$U'(M) := \sum_i [b'_i(M) - c'_i(m_i^m(M)) \cdot (m_i^m)'(M)].$$

With linear-quadratic preferences,  $(m_i^m)'(M) < 1$  for all  $M \geq M^s$ . Furthermore,  $\mathbf{m}^m(M^s) = \mathbf{m}^s$  because the status quo can be implemented as the market equilibrium with price zero. Combined with (2), we get that

$$U'(M^s) > \sum_i [b'_i(M^s) - c'_i(m_i^s)] = 0.$$

In words, countries' aggregate utility goes up when total immigration is slightly raised above the status-quo level. As  $M$  increases further,  $U'(M)$  diminishes because countries' marginal costs grow faster than their marginal benefits. In the linear-quadratic case,  $U'(M) \geq 0$  if and only if  $M \leq M^e$ . By continuity, it follows that utilitarian welfare exceeds the status-quo level even for values of  $M$  slightly above  $M^e$ —which is what [Proposition 4](#) says.

### 3.4 Participation constraints

Even if the market equilibrium raises aggregate utility compared to the status quo, some countries may become worse off. This occurs when their initial quota far exceeds their status-quo immigration level. For these countries, it is not individually rational to sign on to the mechanism. By adjusting the initial quotas, however, the welfare gains can be distributed so that all

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<sup>6</sup>Linear-quadratic specifications are frequently used in environmental economics. See, for instance, Battaglini and Harstad (2016), Holland and Yates (2015), Martimort and Sand-Zantman (2016), or Slechten (2020).

countries participate voluntarily.<sup>7</sup>

**Proposition 5.** *Consider any  $M > M^s$  and suppose that*

$$\sum_i [b_i(M) - c_i(m_i^m(M))] > \sum_i [b_i(M^s) - c_i(m_i^s)].$$

*Then there exists  $\mathbf{m}^0(M) := (m_i^0(M))_{i=1}^n \in \mathbb{R}_+^n$  such that  $\sum_i m_i^0(M) = M$  and*

$$\begin{aligned} \forall i, \quad & b_i(M) - c_i(m_i^m(M)) + p^m(M) [m_i^m(M) - m_i^0(M)] \\ & > b_i(M^s) - c_i(m_i^s). \end{aligned}$$

The intuition behind this result, which we formally prove in [Appendix A.7](#), is simple: Since each country’s payoff function is quasilinear in money, utility is transferable. A one-unit reduction of  $i$ ’s initial quota raises its welfare by  $p^m(M)$ . If lower quotas are assigned to countries that stand to gain less from the market, then all will be better off than in the status quo.

There are some practical caveats. As with [Proposition 3](#), the planner must know the shape of the benefit and cost functions in order to set the initial quotas correctly. Even if this information exists, it may be politically unfeasible to make the initial quotas dependent on preferences. The public may object to rewarding “closed” countries—whose benefits are low and costs are high—with low quotas, whereas “open” countries are punished with high quotas. Further research, incorporating asymmetric information and political constraints, should be devoted to these issues.

### 3.5 Impure public good

So far, we have assumed that countries’ benefits are determined by the global number of immigrants, regardless of their distribution. In practice, a country may derive additional value from offering visas itself—due to, for example, the “warm glow of giving” ([Andreoni, 1990](#)), the gain in humanitarian prestige or the belief that other countries offer worse reception

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<sup>7</sup>FHM-R make this claim as well, but their mathematical argument is incorrect (section 2.1.1, pp. 98–99). According to their equations (16) to (18), the initial quotas must be such that the market equilibrium and the status quo yield the same aggregate utility—which is contradictory. The root of FHM-R’s mistake is maximization problem (14), whose objective function does not depend on the choice variables. Hence, contrary to (15), the multipliers of the constraints must be zero.

conditions. Along these lines, refugee protection has been described as an “impure” public good whose benefits partially accrue to the provider alone (Betts, 2003).

In our model, these “impurities” can be captured by a more general benefit function  $b_i: \mathbb{R}_+^n \rightarrow \mathbb{R}_+$  that depends on the full vector  $\mathbf{m}$  rather than just  $\sum_j m_j$ . FHM-R consider the special case in which each  $b_i$  is additively separable in  $m_i$  and  $\sum_{j \neq i} m_j$ . Formally,  $b_i(\mathbf{m}) = s_i(m_i) + e_i(\sum_{j \neq i} m_j)$ , where  $s_i(m_i) \geq 0$  is  $i$ ’s “self benefit” and  $e_i(\sum_{j \neq i} m_j) \geq 0$  is  $i$ ’s “external benefit”. Additive separability means that  $i$ ’s willingness to host immigrants is independent of what the other countries do.<sup>8</sup>

All our results have natural analogues in FHM-R’s model. In particular, the standard market equilibrium generally fails to be constrained efficient—albeit for a slightly different reason than in [Proposition 1](#) above. In our model, the constrained optimum equalizes marginal costs, whereas the market equilibrium may not. In FHM-R’s model, the market equilibrium equalizes marginal costs, whereas the constrained optimum may not. Despite this difference, the root cause of the inefficiency remains the same: countries act in their own interest, ignoring the positive externality that their own intake of immigrants has on the other countries. Only if we restrict countries’ preferences can we achieve the first best. For example, FHM-R’s [Proposition 1](#) (p. 98) implicitly assumes that the marginal externalities are constant and identical across countries, that is, there exists  $\epsilon > 0$  such that  $e_i(\sum_{j \neq i} m_j) = \epsilon \sum_{j \neq i} m_j$  for all  $i$  and  $\sum_{j \neq i} m_j \geq 0$ . In this case, the constrained optimum equalizes marginal costs and can thus be implemented as the market equilibrium. Another path to efficiency is the alternative, non-Nash, equilibrium concept from above. Our [Proposition 2](#) also holds in FHM-R’s model, as the interested reader may verify.

## 4 Matching

This section extends the baseline model by incorporating migrants’ preferences over host countries. We focus on “one-sided” matching and maintain the assumption that countries view migrants as homogeneous. The case

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<sup>8</sup>In line with our previous assumptions on  $b_i$ , let  $s_i$  and  $e_i$  be strictly increasing, weakly concave and continuously differentiable. Then  $c_i - s_i$  is a strictly convex function with an interior minimum, corresponding to FHM-R’s net cost function  $c_i$ . Moreover, our  $e_i$  is equivalent to their  $g_i$ .

of “two-sided” matching, where countries may prefer some migrants over others, will be discussed in [Section 4.3](#).

The consideration of migrant preferences introduces “matching constraints” that render certain allocations unfeasible. Specifically, some countries may be so unattractive that less than their desired number of migrants are willing to go there. One-sided matching can thus only be detrimental, never beneficial, to host countries. The main reason why it has been advocated in practice is that *migrants’* welfare may improve compared to random assignment. Adopting this view, we redefine efficiency as the utilitarian optimum for host countries that is compatible with migrant preferences. We then ask whether “matching-constrained efficiency” can be achieved via tradable immigration quotas. The answer is generally “no”, even if the market equilibrium is efficient in the absence of matching. The reason is that matching creates further externalities that countries disregard when taking their individually optimal decisions.

#### 4.1 One-sided deterministic matching

Compared to the baseline model, the new ingredient is a vector of “matching functions”  $\boldsymbol{\mu} := (\mu_i)_{i=1}^n$ , which encode both migrant preferences and the matching mechanism. Each  $\mu_i: \mathbb{R}_+^n \rightarrow \mathbb{R}_+$  maps a vector of “capacities”  $\mathbf{q} := (q_i)_{i=1}^n \in \mathbb{R}_+^n$  to the number of migrants matched to country  $i$ ,  $\mu_i(\mathbf{q}) \geq 0$ . The capacities are upper bounds, so  $\mu_i(\mathbf{q}) \leq q_i$ . Subject to these capacities, a one-sided many-to-one matching mechanism assigns migrants to countries. The matching mechanism is deterministic, that is, given capacities  $\mathbf{q}$ , country  $i$  will receive  $\mu_i(\mathbf{q})$  migrants for sure. One example of a deterministic matching mechanism is “simple serial dictatorship”: There is a fixed ordering of migrants on  $\mathbb{R}_+$ . Starting with the migrant ranked first, they sequentially choose their most preferred country among those whose capacities are not yet filled.<sup>9</sup> As we will see in [Section 4.2](#), the results below also carry over to random matching.

We may think of  $\boldsymbol{\mu}$  as the outcome of a centralized matching mechanism run by the planner (e.g. European Commission, UNHCR). While the choice of mechanism may affect countries’ welfare, we take the matching functions

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<sup>9</sup>This description implicitly assumes a discrete number of migrants (“sequentially”). Notwithstanding, serial dictatorship is well defined in continuum economies, such as ours; see Che and Kojima (2010, section 5.2).

$\boldsymbol{\mu}$  as given and thus abstract away from market-design questions. Moreover, migrant preferences plausibly constrain the status quo as well, despite the absence of a centralized matching mechanism. In principle, we could assume a “natural” matching process  $\boldsymbol{\mu}^s$  in the status quo, which is then replaced by a centralized matching mechanism  $\boldsymbol{\mu}$ . This distinction remains largely irrelevant, however, because the status quo plays no role below.

Let us start the analysis with the simple observation that efficiency may be incompatible with the matching constraints.

**Example 3.** There are two countries with matching functions  $\mu_1(\mathbf{q}) = q_1$  and  $\mu_2(\mathbf{q}) = 0$  for all  $\mathbf{q} \in \mathbb{R}_+^2$ . One possible interpretation is that all migrants find country 1 acceptable and country 2 unacceptable. Since the pool of potential migrants has infinite mass, country 1’s capacity will always be filled. Assuming the matching mechanism to be individually rational for migrants, none of them will be sent to country 2.

By Lemma 2, the efficient allocation  $\mathbf{m}^e$  satisfies  $c'_i(m_i^e) = \sum_j b'_j(M^e)$  for both  $i$ . As  $m_2^e > 0 = \mu_2(\mathbf{q})$  for all  $\mathbf{q} \in \mathbb{R}_+^2$ , efficiency is unattainable under matching. Subject to the matching constraints, the two countries’ aggregate utility is maximized by  $\mathbf{m}^{me} = (m_1^{me}, 0)$  with  $c'_1(m_1^{me}) = \sum_j b'_j(m_1^{me})$ . We call this allocation “matching-constrained efficient”.  $\triangle$

**Definition 7.** Given matching functions  $\boldsymbol{\mu}$ , an allocation  $\mathbf{m}^{me} := (m_i^{me})_{i=1}^n \in \mathbb{R}_+^n$  is *matching-constrained efficient*, or a *matching-constrained optimum*, if

$$\begin{aligned} \mathbf{m}^{me} \in \arg \max_{\mathbf{m} \in \mathbb{R}_+^n} \sum_i \left[ b_i \left( \sum_j m_j \right) - c_i(m_i) \right] \\ \text{s.t. } \exists \mathbf{q} \in \mathbb{R}_+^n : \forall i, \mu_i(\mathbf{q}) = m_i. \end{aligned} \quad (15)$$

Without further conditions on  $\boldsymbol{\mu}$ , (15) may not have a solution. But assuming that a matching-constrained optimum  $\mathbf{m}^{me}$  exists, can it be implemented via tradable quotas? Define  $M^{me} := \sum_i m_i^{me}$  and suppose that each country  $i$  is assigned an initial capacity  $q_i^0 \geq 0$  such that  $\sum_i \mu_i(\mathbf{q}^0) = M^{me}$ . These capacities can be traded in a competitive market. The resulting vector  $\mathbf{q}^{mm} \in \mathbb{R}_+^n$  determines the capacities in the matching mechanism. When deciding how much to trade, each country must take into account that changing its capacity may affect the outcome of the matching mechanism, both for itself and for the other countries.

**Definition 8.** Consider any  $\mathbf{q}^0 := (q_i^0)_{i=1}^n \in \mathbb{R}_+^n$  such that  $\sum_i \mu_i(\mathbf{q}^0) = M^{me}$ . A *matching-constrained market equilibrium*  $(\mathbf{q}^{mm}; p^{mm})$  consists of capacities  $\mathbf{q}^{mm} := (q_i^{mm})_{i=1}^n \in \mathbb{R}_+^n$  and a price  $p^{mm} \geq 0$  such that

$$\forall i, \quad q_i^{mm} \in \arg \max_{q_i \geq 0} \left\{ b_i \left( \sum_j \mu_j(q_i, \mathbf{q}_{-i}^{mm}) \right) - c_i(\mu_i(q_i, \mathbf{q}_{-i}^{mm})) \right. \\ \left. + p^{mm} [\mu_i(q_i, \mathbf{q}_{-i}^{mm}) - \mu_i(\mathbf{q}^0)] \right\} \quad (16)$$

and  $\sum_i \mu_i(\mathbf{q}^{mm}) = M^{me}$ .

Note that the market price  $p^{mm}$  applies to the change in the post-matching immigration levels (i.e.  $\mu_i(\mathbf{q}^{mm}) - \mu_i(\mathbf{q}^0)$ ), not to the change in capacities (i.e.  $q_i^{mm} - q_i^0$ ). Otherwise, an unattractive country may want to increase its capacity infinitely much, knowing that the available slots will not be filled (see FHM-R, p. 100).

In [Example 3](#) above, the matching-constrained optimum can be implemented as a matching-constrained market equilibrium.

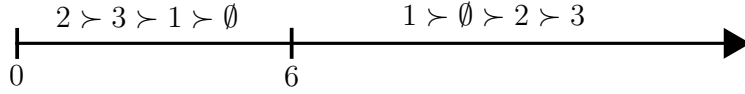
**Example 3** (continued). Since  $\mu_2(\mathbf{q}) = 0$  for all  $\mathbf{q} \in \mathbb{R}_+^2$ , country 2 will never receive any migrants, regardless of its capacity. Thus, any  $q_2^{mm} \geq 0$  solves (16) for  $i = 2$ . Moreover, given that  $\mu_1(\mathbf{q}) = q_1$  for all  $\mathbf{q} \in \mathbb{R}_+^2$ , (16) for  $i = 1$  simplifies to

$$q_1^{mm} \in \arg \max_{q_1 \geq 0} \{ b_1(q_1) - c_1(q_1) + p^{mm}(q_1 - q_1^0) \}.$$

The unique solution satisfies  $c_1'(q_1^{mm}) - b_1'(q_1^{mm}) = p^{mm}$ . By market clearing,  $q_1^{mm} = m_1^{me}$ . Recall from above that  $c_1'(m_1^{me}) - b_1'(m_1^{me}) = b_2'(m_1^{me})$ . Hence,  $p^{mm} = b_2'(m_1^{me})$ . To conclude, the set of matching-constrained market equilibria is characterized by  $q_1^{mm} = m_1^{me}$ ,  $q_2^{mm} \geq 0$  and  $p^{mm} = b_2'(m_1^{me})$ . They are all matching-constrained efficient because  $\boldsymbol{\mu}(\mathbf{q}^{mm}) = \mathbf{m}^{me}$ .  $\triangle$

[Example 3](#) is special in that each country's capacity only affects its own level of immigration, not that of the other country. In general, the matching mechanism may introduce externalities because an attractive country that expands its capacity may divert migrants away from less popular destinations. The social planner internalizes these externalities, but individual countries do not. For this reason, matching-constrained market equilibria may fail to be matching-constrained efficient.

**Example 4.** There are three countries with identical preferences:  $b_i(\sum_j m_j) = 2\sum_j m_j$  and  $c_i(m_i) = \frac{1}{2}(m_i)^2$  for all  $i$  and  $\mathbf{m} := (m_i)_{i=1}^3 \in \mathbb{R}_+^3$ . The matching functions are given by  $\mu_1(\mathbf{q}) = q_1$ ,  $\mu_2(\mathbf{q}) = \min\{q_2, 6\}$  and  $\mu_3(\mathbf{q}) = \min\{q_3, \max\{6 - q_2, 0\}\}$  for all  $\mathbf{q} := (q_i)_{i=1}^3 \in \mathbb{R}_+^3$ . Figure 1 illustrates one possible interpretation of  $\boldsymbol{\mu} := (\mu)_{i=1}^3$ : Matching is by simple serial dictatorship. First in line are migrants of (Lebesgue) measure 6 who find all countries acceptable and prefer 2 to 3 to 1. All other migrants—a set of infinite measure—are only willing to go to country 1, not to 2 or 3.



**Figure 1:** Illustration of the matching functions in Example 4

*Note:* Matching is by simple serial dictatorship. Migrants are ordered on the non-negative real line. Each migrant in the set with Lebesgue measure 6 ranked at the top (i.e. 0) prefers country 2 to country 3, country 3 to country 1, and country 1 to being unmatched ( $2 \succ 3 \succ 1 \succ \emptyset$ ). All other migrants prefer country 1 to being unmatched, being unmatched to country 2, and country 2 to country 3 ( $1 \succ \emptyset \succ 2 \succ 3$ ).

Without matching, efficiency requires that  $c'_i(m_i^e) = \sum_j b'_j(M^e)$  for all  $i$ , yielding  $\mathbf{m}^e = (6, 6, 6)$ . This allocation is incompatible with the matching mechanism because, for all  $\mathbf{q} \in \mathbb{R}_+^3$ ,  $\mu_2(\mathbf{q}) = m_2^e$  implies that  $q_2 \geq 6$  and thus  $\mu_3(\mathbf{q}) = 0 < m_3^e$ . In words, no migrants are willing to go to country 3 because all those who find it acceptable are wooed away by country 2.

As we formally show in Appendix A.8.1, the matching-constrained optimum is  $\mathbf{m}^{me} = (6, 3, 3)$ . Since country 1 neither constrains the other two countries nor is constrained by them,  $m_1^{me} = m_1^e$ . By contrast, countries 2 and 3 receive the largest symmetric immigration levels compatible with the matching constraint that 2 imposes on 3. When  $m_2^{me} = m_3^{me} = 3$ , any increase in  $m_2$  would cause a one-for-one decrease in  $m_3$ . The total benefit from immigration would remain unchanged, while the total cost would increase, thus reducing utilitarian welfare.

Appendix A.8.2 characterizes the continuum of matching-constrained market equilibria. They all satisfy  $\mu_1(\mathbf{q}^{mm}) = 6$ ,  $\mu_2(\mathbf{q}^{mm}) \in [4, 6]$ ,  $\mu_3(\mathbf{q}^{mm}) = 6 - \mu_2(\mathbf{q}^{mm})$  and  $p^{mm} = 4$ . Since  $\mu_2(\mathbf{q}^{mm}) \geq 4 > 3 = m_2^{me}$ , country 2 will take in more migrants than is matching-constrained efficient. Conversely, country 2 will host too few migrants because  $\mu_3(\mathbf{q}^{mm}) \leq 2 < 3 = m_3^{me}$ . To grasp the intuition, note first that country 1 optimally accepts six migrants

at price  $p^{mm} = 4$ . Country 2 may wish to have fewer than six because its benefit from extending capacity is offset by an equal decrease in country 3's immigration level. What country 2 ignores, however, is that raising its own capacity will reduce country 3's utility, by pushing down the latter's immigration level below its optimum of six. For this reason, the matching-constrained market equilibrium will assign too many immigrants to country 2, and too few to country 3.  $\triangle$

By [Definitions 2](#) and [7](#), the matching-constrained optimum coincides with the unconstrained optimum if  $\mu_i(\mathbf{q}) = q_i$  for all  $i$  and  $\mathbf{q} \in \mathbb{R}_+^n$ . The same is true for the matching-constrained market equilibrium ([Definitions 4](#) and [8](#)). In this case, the conclusion that the matching-constrained market equilibrium may fail to be matching-constrained efficient trivially follows from [Proposition 1](#). What gives [Example 4](#) bite is the assumption that the benefit functions are identical and linear. Hence, by [Proposition 1](#), the market equilibrium is efficient in the absence of matching.

One reason for the market failure in [Example 4](#) is that few migrants are willing to go to country 3. In practice, however, rich countries may be able to attract enough migrants for any number of visas offered. For example, most Syrian refugees would probably rather be resettled to *any* EU member state than stay in squalid tent settlements in Lebanon ([Karasapan & Shah, 2021](#)). If each country is acceptable to an infinite mass of migrants, then the matching externalities vanish. To state this observation formally, let  $\ell$  denote the Lebesgue measure on the real line, and let  $A_i \subseteq \mathbb{R}_+$  be the set of migrants that find country  $i$  acceptable. A matching mechanism is said to be “non-wasteful” if, for all  $i$  and  $\mathbf{q} \in \mathbb{R}_+^n$ ,  $\mu_i(\mathbf{q}) < q_i$  implies that almost all migrants in  $A_i$  are hosted by some country; otherwise both  $i$  and the unassigned migrants in  $A_i$  would benefit from being matched to each other.

**Proposition 6.** *Suppose that  $\ell(A_i) = \infty$  for all  $i$ , and that the matching mechanism is non-wasteful. Then  $\mu_i(\mathbf{q}) = q_i$  for all  $i$  and  $\mathbf{q} \in \mathbb{R}_+^n$ .*

*Proof.* Consider any  $i$  and  $\mathbf{q} \in \mathbb{R}_+^n$ . If  $\ell(A_i) = \infty$ , then  $\ell(A_i) - \sum_{j \neq i} q_j = \infty$ . This means that, even if the capacities of the other countries are filled entirely with migrants from  $A_i$ , there remains an infinite measure of migrants willing to go to  $i$ . Hence, by non-wastefulness,  $\mu_i(\mathbf{q}) = q_i$ .  $\square$

[Propositions 1](#) and [6](#) combined provide sufficient conditions under which the matching-constrained market equilibrium is efficient. In loose terms, each

country must equally benefit from immigration and be acceptable to a large number of potential migrants.

## 4.2 One-sided random matching

So far, we have assumed that the matching mechanism is deterministic. In simple serial dictatorship, for example, the ordering of migrants is known before countries set their capacities. But what if migrants are randomly ordered *after* the market has been run? We now show that both [Example 4](#) and [Proposition 6](#) remain valid.

To accommodate random matching, we must reinterpret the model. Specifically,  $\mu_i(\mathbf{q})$  now denotes the *expected* number of migrants matched to country  $i$  when the capacities are  $\mathbf{q}$ . The expectations are taken over the randomization device used in the matching mechanism, so they are the same for everyone. Suppose that countries care only about the expected number of migrants rather than the full probability distribution. Then country  $i$ 's expected utility without monetary transfers is  $b_i(\sum_j \mu_j(\mathbf{q})) - c_i(\mu_i(\mathbf{q}))$ , as before. [Definitions 7](#) and [8](#) still apply, except that both  $\mathbf{m}^{me}$  and market clearing now refer to expectations.

[Example 4](#) extends to *random* serial dictatorship, where migrants are ordered by lottery after the capacities are set (cf. Abdulkadiroğlu & Sönmez, 1998). To see why, recall that only a measure six of migrants find countries 2 and 3 acceptable. Moreover, all of these migrants prefer 2 to 3. Hence, the matching externality is present regardless of how migrants are ordered: an increase in country 2's capacity reduces, one for one, the maximum number of migrants available to country 3. It follows that the matching functions  $\mu$  are the same for simple and random serial dictatorship, so the analysis in [Example 4](#) goes through unchanged. In particular, the matching-constrained market equilibria are not matching-constrained efficient.

[Proposition 6](#) also holds under random matching. If each country is acceptable to an infinite mass of migrants, then any non-wasteful matching mechanism—deterministic or random—will fill each country's capacity to the brim.

### 4.3 Two-sided matching

In the wake of Europe’s “refugee crisis” around 2015, various scholars have advocated matching refugees to countries or lower administrative levels (e.g. Jones & Teytelboym, 2016, 2017a, 2017b, 2018). Most proposals endorse two-sided matching mechanisms that consider the preferences of both migrants and countries over each other. In the existing literature, countries are usually assumed to have exogenously given capacities for immigrants. We now briefly discuss how two-sided matching could be incorporated into our model, where countries strategically choose their capacities.

Two-sided matching is only meaningful if countries care not just about *how many* but also about *which* migrants they receive. To capture this scenario in our model, the cost and benefit functions must be adjusted. One possible assumption is that each country’s benefit is still determined by the global level of immigration but its cost now depends on the “quality” of its immigration cohort. For example, migrants who speak the local language may be less costly to host than those who do not.

There are different ways to formally model countries’ preferences over heterogeneous immigrants. FHM-R (section 2.3, pp. 101–104) assume a finite number of migrant types. Each country is indifferent between migrants of the same type, so its cost becomes a multivariate function of its type-specific immigration levels. The tradable-quotas-and-matching system works as before: Countries are assigned initial capacities, one per migrant type, and can trade them in a competitive market. Subject to the type-specific capacities that result from the market, migrants are then matched to countries. Given that countries are indifferent between migrants of the same type, it seems natural to run a one-sided matching mechanism (e.g. serial dictatorship) separately for each type. Instead, FHM-R argue for a two-sided matching mechanism, namely country-proposing deferred acceptance. They do not clarify, however, why and how to apply this mechanism here. The issue is that, in contrast to standard two-sided many-to-one matching models (e.g. college admission), each country has *type-specific* capacities and is *indifferent* between migrants of the same type.

Regardless of the particular modeling approach, the matching externalities continue to exist: a country that extends its capacity may lure migrants away from other countries. In contrast to one-sided matching, certain mi-

grants are now likely to be scarce (e.g. PhD-level engineers). Assuming that there are “enough for everyone”, as we did in [Proposition 6](#), does therefore not appear justified anymore.

Two-sided matching introduces yet another source of inefficiency: when countries care about the composition of the immigrants they receive, changing a country’s capacity may exert not only a *quantitative* but also a *qualitative* externality on the other countries. For example, when country  $i$  reduces its capacity, some of the immigrants that it would have been assigned lose their slot and apply elsewhere. This may trigger a “rejection chain” because other countries may accept some of the migrants coming from  $i$  and, not to exceed capacity, reject some of their previous matches. Eventually,  $i$  may receive new applicants that it prefers to those who were initially given the boot. Even though  $i$  will end up with less immigrants, those it hosts may be of higher “quality”. What country  $i$  ignores, however, is the change in the other countries’ welfare when it poaches immigrants from them.

The incentives to shade capacity are well-understood in two-sided many-to-one matching mechanisms with *exogenous* capacities and a *finite* number of agents on both sides of the market (Sönmez, 1997).<sup>10</sup> Azevedo (2014) has extended the analysis to a model in which, similar to ours, expanding capacity is costly for firms/countries and there is a continuum of workers/migrants.<sup>11</sup> His study builds on Azevedo and Leshno (2016)’s price-theoretic reformulation of two-sided many-to-one matching with a finite number of agents on one side (countries) and a continuum of agents on the other side (migrants). Along the lines of Azevedo (2014), the integration of two-sided matching into our model may be tractable. Whether such an analysis yields new insights, beyond those pointed out above, is an exciting question for future research.

## 5 Conclusion

We have revisited the theory behind tradable immigration quotas, plus matching, as means to issue more visas to people in need. One take-away is that this mechanism may fail to achieve the first best because countries ignore the positive externalities that they impose on each other when hosting

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<sup>10</sup>See also Ehlers (2010), Kesten (2012), Konishi and Ünver (2006), Mumcu and Saglam (2009), Romero-Medina and Triossi (2013), and Romm (2014).

<sup>11</sup>See also Honarvar and Shahdadi (2021).

more immigrants. Notwithstanding, we have presented several theoretical arguments in favor of tradable immigration quotas. From a practical perspective, the proposal remains relevant as well. It offers a novel approach to low-skilled South-North migration and, specifically, EU asylum policy. As mentioned in the [Introduction](#), Fernández-Huertas Moraga and Rapoport encountered two main reservations when presenting their idea to politicians in Europe. To conclude this paper, we sketch how future research could address both points of criticism.

First, national governments objected to a matching mechanism that incorporates asylum seekers' preferences over destination countries, for fear of creating further incentives to set out for Europe. This pull effect does not occur in our model because there is a fixed pool of potential migrants, and countries can limit the number of people they let in. How to design matching mechanisms that give migrants more of a say without raising migratory pressure merits theoretical investigation.

Second, there were ethical concerns about the “commodification” of immigrants. The buying and selling of asylum responsibilities is widely perceived as repugnant and thus politically unfeasible (cf. Roth, 2007). Whether markets indeed demean refugees is the subject of an ongoing philosophical debate (Gerver, 2018; Himmelreich, 2019; Kuosmanen, 2013). Based on this literature, it appears worthwhile to develop a formal definition of morally acceptable mechanisms and analyze whether they can allocate (forced) migrants more efficiently than non-market instruments.<sup>12</sup>

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<sup>12</sup>For existing work on market design under ethical constraints, see Huesmann and Wambach (2015) and Li (2017).

## A Appendix

### A.1 Proof of Lemma 1

At the solution to (1), the non-negativity constraint on  $m_i$  will not bind because  $b'_i(0 + \sum_{j \neq i} m_j^s) - c'_i(0) = b'_i(\sum_{j \neq i} m_j^s) > 0$  for any  $\sum_{j \neq i} m_j^s \geq 0$ . Hence, (1) is effectively an unconstrained maximization problem. Since the objective function is continuously differentiable and strictly concave, the first-order conditions are necessary and sufficient. They read

$$\forall i, \quad c'_i(m_i^s) = b'_i\left(\sum_j m_j^s\right). \quad (\text{A.1})$$

Since  $c_i$  is strictly convex,  $c'_i$  has an inverse, which we denote by  $(c'_i)^{-1}$ . Defining  $M^s := \sum_j m_j^s$  and summing (A.1) over all  $i$ , we obtain that

$$M^s = \sum_i (c'_i)^{-1}(b'_i(M^s)). \quad (\text{A.2})$$

We now show that there exists a unique  $M^s > 0$  that satisfies (A.2). Define a function  $a: \mathbb{R}_+ \rightarrow \mathbb{R}$  by  $a(x) := x - \sum_i (c'_i)^{-1}(b'_i(x))$  for all  $x \geq 0$ . Since  $c'_i$  is continuous and strictly increasing,  $(c'_i)^{-1}$  is as well. Also,  $b'_i$  is continuous and weakly decreasing. It follows that  $a$  is continuous and strictly increasing. Moreover,  $a(0) = 0 - \sum_i (c'_i)^{-1}(b'_i(0)) < 0$  and  $\lim_{x \rightarrow \infty} a(x) = \infty$ . Hence, by Bolzano's Theorem, there exists a unique  $x > 0$  such that  $a(x) = 0$ . We conclude that (A.2) has a unique solution. Since  $c'_i$  is continuous and strictly increasing, (A.1) implies that  $m_i^s$  is unique as well.

### A.2 Proof of Lemma 2

Closely following the proof of Lemma 1, it is easy to verify that  $\mathbf{m}^e$  exists, is unique and satisfies (4). We omit the details and merely show that  $M^e := \sum_i m_i^e > M^s$ . To the contrary, suppose that  $M^e \leq M^s$ . Then, by (2) and (4),

$$\forall i, \quad c'_i(m_i^e) = \sum_j b'_j(M^e) \geq \sum_j b'_j(M^s) > b'_i(M^s) = c'_i(m_i^s).$$

Since  $c'_i$  is strictly increasing, it follows that  $m_i^e > m_i^s$  for all  $i$ . Thus,  $M^e = \sum_i m_i^e > \sum_i m_i^s = M^s$ , a contradiction.

### A.3 Proof of Lemma 3

Consider any  $M \geq M^s$ . By Lemma 1,  $M^s > 0$  and thus  $M > 0$ .

The objective function in (5),  $\sum_i [b_i(\sum_j m_j) - c_i(m_i)]$ , is continuous and strictly concave in  $\mathbf{m}$ . The choice set,  $\{\mathbf{m} \in \mathbb{R}_+^n : \sum_i m_i \leq M\}$ , is compact and convex in  $\mathbf{m}$ . Hence, there exists a unique  $\mathbf{m}^c(M)$  that solves (5).

If  $M \geq M^e$ , then  $\mathbf{m}^e$  is in the choice set, so  $\mathbf{m}^c(M) = \mathbf{m}^e$ . Clearly,  $\sum_i m_i^c(M) = \sum_i m_i^e = M^e = \min\{M, M^e\}$ . Moreover, (4) implies (6).

Suppose now that  $M < M^e$ . The Kuhn-Tucker conditions are necessary and sufficient because the objective function is continuously differentiable and concave, while the choice set is defined by linear constraints. Let  $\Lambda(M) \geq 0$  and  $\lambda_i(M) \geq 0$  be the multipliers of the constraints  $M - \sum_i m_i \geq 0$  and  $m_i \geq 0$ , respectively. Define  $M^c(M) := \sum_i m_i^c(M)$ . The first-order conditions of (5) read

$$\forall i, \quad \sum_j b'_j(M^c(M)) - c'_i(m_i^c(M)) - \Lambda(M) + \lambda_i(M) = 0. \quad (\text{A.3})$$

Complementary slackness requires that

$$\Lambda(M)[M - M^c(M)] = 0 \quad (\text{A.4})$$

and

$$\forall i, \quad \lambda_i(M)m_i^c(M) = 0. \quad (\text{A.5})$$

Let us show that  $\lambda_i(M) = 0$  for all  $i$ . To the contrary, suppose there exists  $i$  with  $\lambda_i(M) > 0$ . By (A.5),  $m_i^c(M) = 0$  and thus  $c'_i(m_i^c(M)) = 0$ . If  $\Lambda(M) = 0$ , then (A.3) implies that  $\sum_j b'_j(M^c(M)) = -\lambda_i(M) < 0$ , which violates the assumption that each  $b_j$  is strictly increasing. Hence,  $\Lambda(M) > 0$ . By (A.4),  $M^c(M) = M > 0$ . Thus, there exists  $k \neq i$  with  $m_k^c(M) > 0$  and, by (A.5),  $\lambda_k(M) = 0$ . Applying (A.3) to both  $k$  and  $i$ , we get that

$$c'_k(m_k^c(M)) = \sum_j b'_j(M^c(M)) - \Lambda(M) = -\lambda_i(M) < 0,$$

which violates the assumption that  $c_k$  is strictly increasing.

Since  $\lambda_i(M) = 0$  for all  $i$ , (A.3) becomes

$$\forall i, \quad c'_i(m_i^c(M)) = \sum_j b'_j(M^c(M)) - \Lambda(M). \quad (\text{A.6})$$

The right-hand side is the same for all  $i$ , so (A.6) implies (6).

Finally, we show that  $M^c(M) = M$ . If  $M^c(M) < M$ , then (A.4) implies that  $\Lambda(M) = 0$ . Hence, (A.6) becomes equivalent to (4). Since the unique solution to (4) is  $\mathbf{m}^e$ , we have that  $\mathbf{m}^c(M) = \mathbf{m}^e$ . But then  $M^c(M) = M^e > M$ , a contradiction.

#### A.4 Proof of Lemma 4

The proof is similar to the one of Lemma 1. Consider any  $M \geq M^s$ . To simplify notation, let us suppress the dependence of  $(\mathbf{m}^m; p^m)$  on  $M$ .

The constraint  $m_i \geq 0$  in (7) can be ignored because  $b'_i(0 + \sum_{j \neq i} m_j^m) - c'_i(0) + p^m > 0$  for any  $\sum_{j \neq i} m_j^m \geq 0$  and  $p^m \geq 0$ , implying that  $m_i^m > 0$ . Since the objective function is continuously differentiable and strictly concave, the first-order conditions are necessary and sufficient for the solution to (7). They read

$$\forall i, \quad c'_i(m_i^m) = b'_i\left(\sum_j m_j^m\right) + p^m. \quad (\text{A.7})$$

Since  $c_i$  is strictly convex,  $c'_i$  has an inverse  $(c'_i)^{-1}$ . Summing (A.7) over all  $i$  and using market clearing (i.e.  $\sum_j m_j^m = M$ ), we obtain that

$$M = \sum_i (c'_i)^{-1}(b'_i(M) + p^m). \quad (\text{A.8})$$

We now show that there exists a unique  $p^m \geq 0$  that satisfies (A.8). Define a function  $z: \mathbb{R}_+^2 \rightarrow \mathbb{R}$  by  $z(x, y) := \sum_i (c'_i)^{-1}(b'_i(x) + y) - x$  for all  $x, y \geq 0$ . The value  $z(x, y)$  can be interpreted as the excess demand when a total of  $x$  quotas are available at price  $y$ . Given our assumptions on  $\{b_i, c_i\}_{i=1}^n$ ,  $z(\cdot, y)$  is strictly decreasing for all  $y \geq 0$ . Moreover,  $z(x, \cdot)$  is strictly increasing and continuous for all  $x \geq 0$ . From equation (A.2) in the proof of Lemma 1, we know that  $z(M^s, 0) = 0$ . As  $z(M^s, \cdot)$  is strictly increasing,  $z(M^s, y) > 0$  for all  $y > 0$ . Hence, if  $M = M^s$ , then  $p^m = 0$  is the unique price that solves (A.8). Now consider any  $M > M^s$ . Since  $z(M^s, 0) = 0$  and  $z(\cdot, 0)$  is strictly decreasing,  $z(M, 0) < 0$ . Also, by assumption,  $\lim_{m_i \rightarrow \infty} c'_i(m_i) = \infty$  for all  $i$ , so  $\lim_{y \rightarrow \infty} z(M, y) = [\lim_{y \rightarrow \infty} \sum_i (c'_i)^{-1}(b'_i(M) + y)] - M = \infty$ . As  $z(M, \cdot)$  is strictly increasing and continuous, Bolzano's Theorem assures the existence of a unique  $y > 0$  such that  $z(M, y) = 0$ . Hence,  $p^m$  is unique. Since  $c'_i$  is continuous and strictly increasing, (A.7) implies that  $m_i^m$  is unique as well.

## A.5 Proof of Lemma 5

The Kuhn-Tucker conditions are necessary and sufficient for the solution to (9). Suppressing notation of  $M$ , they include

$$\forall i, \quad c'(m_i^a) = p^a + \lambda_i, \quad (\text{A.9})$$

where  $\lambda_i \geq 0$  is the multiplier of the constraint  $m_i \geq 0$ . If  $\lambda_i > 0$ , then complementary slackness requires that  $m_i^a = 0$  and thus  $c'_i(m_i^a) = 0$ . Since  $p^a \geq 0$ , (A.9) is violated. Therefore,  $\lambda_i = 0$ . Given that  $c'_i$  is invertible, (A.9) equivalently reads

$$\forall i, \quad m_i^a = (c'_i)^{-1}(p^a). \quad (\text{A.10})$$

Sum over all countries and use market clearing to get  $M = \sum_i (c'_i)^{-1}(p^a)$ . Applying the argument from the proof of Lemma 4, this equation holds for a unique  $p^a > 0$ . By (A.10),  $m_i^a$  is also unique and satisfies  $c'_i(m_i^a) = p^a$ .

## A.6 Proof of Proposition 4

For all  $M \geq M^s$ , define  $U(M) := \sum_i [b_i(M) - c_i(m_i^m(M))]$ . Then

$$\begin{aligned} U'(M) &= \sum_i \left[ b'_i(M) - \underbrace{c'_i(m_i^m(M))}_{=b'_i(M)+p^m(M)} (m_i^m)'(M) \right] \\ &= \sum_i b'_i(M) [1 - (m_i^m)'(M)] - p^m(M) \underbrace{\sum_i (m_i^m)'(M)}_{=1}. \end{aligned} \quad (\text{A.11})$$

Suppose that, for all  $i$ , there exist  $\beta_i > 0$  and  $\gamma_i > 0$  such that  $b_i(\sum_j m_j) = \beta_i \sum_j m_j$  and  $c_i(m_i) = \frac{1}{2} \gamma_i (m_i)^2$  for all  $\mathbf{m} \in \mathbb{R}_+^n$ . Then (8) becomes

$$\forall i, \quad m_i^m(M) = \frac{\beta_i + p^m(M)}{\gamma_i}.$$

Combined with  $\sum_i m_i^m(M) = M$ , we get that

$$p^m(M) = \frac{M - \sum_i (\beta_i / \gamma_i)}{\sum_i (1 / \gamma_i)}.$$

Hence,

$$\forall i, \quad (m_i^m)'(M) = \frac{(p^m)'(M)}{\gamma_i} = \frac{1 / \gamma_i}{\sum_j (1 / \gamma_j)}.$$

Plug into (A.11) to obtain that

$$\begin{aligned}
U'(M) &= \sum_i \beta_i \left[ 1 - \frac{1/\gamma_i}{\sum_j (1/\gamma_j)} \right] - \frac{M - \sum_i (\beta_i/\gamma_i)}{\sum_i (1/\gamma_i)} \\
&= \frac{\sum_i \beta_i \sum_{j \neq i} (1/\gamma_j)}{\sum_j (1/\gamma_j)} - \frac{M - \sum_i (\beta_i/\gamma_i)}{\sum_i (1/\gamma_i)} \\
&= \frac{\sum_i (1/\gamma_i) \sum_{j \neq i} \beta_j}{\sum_i (1/\gamma_i)} - \frac{M - \sum_i (1/\gamma_i) \beta_i}{\sum_i (1/\gamma_i)} \\
&= \frac{\sum_i (1/\gamma_i) \sum_j \beta_j - M}{\sum_i (1/\gamma_i)} = \frac{M^e - M}{\sum_i (1/\gamma_i)},
\end{aligned}$$

where the last equality follows from  $m_i^e = (1/\gamma_i) \sum_j \beta_j$  for all  $i$ . Integrating  $U'$ , it follows that  $U(M) > U(M^s)$  for all  $M \in (M^s, M^e]$ . Since  $U$  is continuous, there exists  $\bar{M} > M^e$  such that  $U(M) > U(M^s)$  for all  $M \in (M^s, \bar{M})$ .

## A.7 Proof of Proposition 5

Consider any  $M > M^s$  and let us suppress its notation. For all  $i$ , define

$$\Delta u_i := [b_i(M) - c_i(m_i^m)] - [b_i(M^s) - c_i(m_i^s)]. \quad (\text{A.12})$$

By hypothesis,  $\Delta U := \sum_i \Delta u_i > 0$ . Set the initial quotas as follows:

$$\forall i, \quad m_i^0 := m_i^m + \frac{\Delta u_i - \Delta U/n}{p^m}. \quad (\text{A.13})$$

Summing over all countries yields that  $\sum_i m_i^0 = \sum_i m_i^m (= M)$ , as required. By (A.12) and (A.13), the market equilibrium increases each country's utility by  $\Delta U/n$  compared to the status quo:

$$\begin{aligned}
\forall i, \quad & b_i(M) - c_i(m_i^m) + p^m(m_i^m - m_i^0) \\
&= b_i(M) - c_i(m_i^m) - \Delta u_i + \Delta U/n \\
&= b_i(M^s) - c_i(m_i^s) + \Delta U/n \\
&> b_i(M^s) - c_i(m_i^s).
\end{aligned}$$

## A.8 Proof of Example 4

This section characterizes the matching-constrained optimum ( $\mathbf{m}^{me}$ ) and the matching-constrained market equilibria ( $\mathbf{q}^{mm}; p^{mm}$ ) in Example 4.

### A.8.1 Matching-constrained optimum

First, we show that  $\mathbf{m}^{me} = (6, 3, 3)$  is the matching-constrained optimum.

Since  $\mu_1(\mathbf{q}) = q_1$  for all  $\mathbf{q} \in \mathbb{R}_+^3$ , and since  $q_1$  affects neither  $\mu_2$  nor  $\mu_3$ , the matching mechanism is irrelevant for country 1. Applying (15),

$$m_1^{me} \in \arg \max_{m_1 \geq 0} \{6m_1 - \frac{1}{2}(m_1)^2\} \iff m_1^{me} = 6.$$

To determine  $m_2^{me}$  and  $m_3^{me}$ , we study the underlying capacities. For all  $\mathbf{q} \in \mathbb{R}_+^3$  with  $q_2 \geq 6$ ,  $\mu_2(\mathbf{q}) = 6$  and  $\mu_3(\mathbf{q}) = 0$ . Since they are constant, it is without loss of generality to assume that  $q_2 \leq 6$ . The matching functions then simplify to  $\mu_2(\mathbf{q}) = q_2$  and  $\mu_3(\mathbf{q}) = \min\{q_3, 6 - q_2\}$ . Country 3 does not constrain the other two countries but faces an upper bound of  $6 - q_2$ . Absent this constraint, efficiency would require that  $q_3 = 6$ , as for country 1. Given the upper bound and the strict concavity of the objective function, any  $q_3 \geq 6 - q_2$  is optimal, so country 3's matching function becomes  $\mu_3(\mathbf{q}) = 6 - q_2$ . An increase in  $q_2$  will raise  $\mu_2(\mathbf{q}) = q_2$  and lower  $\mu_3(\mathbf{q}) = 6 - q_2$  by the same amount, keeping the aggregate benefit from immigration unchanged. Hence, the matching-constrained efficient  $q_2 \in [0, 6]$  minimizes  $c_2(q_2) + c_3(6 - q_2)$ . Since  $c_2 = c_3$ , we get  $q_2 = 3$  and thus  $m_2^{me} = m_3^{me} = 3$ .

### A.8.2 Matching-constrained market equilibrium

Second, we show that  $(\mathbf{q}^{mm}; p^{mm})$  is a matching-constrained market equilibrium if and only if  $p^{mm} = 4$ ,  $q_1^{mm} = 6$ ,  $q_3^{mm} \geq 0$  and

$$\begin{aligned} q_2^{mm} &\geq 6 && \text{if } q_3^{mm} = 0, \\ q_2^{mm} &= 6 - q_3^{mm} && \text{if } 0 < q_3^{mm} < 2, \\ q_2^{mm} &= 4 && \text{if } 2 \leq q_3^{mm}. \end{aligned}$$

Since country 1 is unaffected by the matching mechanism, (16) for  $i = 1$  simplifies to

$$q_1^{mm} \in \arg \max_{q_1 \geq 0} \{2q_1 - \frac{1}{2}(q_1)^2 + p^{mm} q_1\} \iff q_1^{mm} = 2 + p^{mm}.$$

For all  $\mathbf{q} \in \mathbb{R}_+^3$  with  $q_2 \geq 6$ ,  $\mu_2(\mathbf{q}) = 6$  and  $\mu_3(\mathbf{q}) = 0$ . Thus, any

matching-constrained market equilibrium with  $q_2^{mm} = 6$  remains such if  $q_2^{mm} > 6$ , and vice versa. With this in mind, we can focus on  $q_2^{mm} \leq 6$ . Then  $\mu_2(\mathbf{q}^{mm}) = q_2^{mm}$  and  $\mu_3(q_3, \mathbf{q}_{-3}^{mm}) = \min\{q_3, 6 - q_2^{mm}\}$ . Absent the upper bound of  $6 - q_2^{mm}$ , country 3 would choose  $2 + p^{mm}$ , just like country 1. Since country 3's objective function is strictly concave in  $q_3$  for  $q_3 \leq 6 - q_2^{mm}$ , and flat for all  $q_3 \geq 6 - q_2^{mm}$ , we have that

$$\begin{aligned} q_3^{mm} &= 2 + p^{mm} & \text{if } 2 + p^{mm} < 6 - q_2^{mm}, \\ q_3^{mm} &\geq 6 - q_2^{mm} & \text{if } 2 + p^{mm} \geq 6 - q_2^{mm}. \end{aligned}$$

Next, we show that  $p^{mm} = 4$ . If  $p^{mm} > 4$ , then  $2 + p^{mm} > 6 \geq 6 - q_2^{mm}$ , implying that  $q_3^{mm} \geq 6 - q_2^{mm}$  and  $\mu_3(\mathbf{q}^{mm}) = 6 - q_2^{mm}$ . Hence,

$$\begin{aligned} \sum_i \mu_i(\mathbf{q}^{mm}) &= (2 + p^{mm}) + q_2^{mm} + (6 - q_2^{mm}) \\ &= 8 + p^{mm} > 8 + 4 = 12 = M^{me}, \end{aligned}$$

which violates market clearing. If  $p^{mm} < 4$ , then

$$\begin{aligned} \sum_i \mu_i(\mathbf{q}^{mm}) &= (2 + p^{mm}) + q_2^{mm} + \min\{q_3^{mm}, 6 - q_2^{mm}\} \\ &\leq (2 + p^{mm}) + q_2^{mm} + (6 - q_2^{mm}) = 8 + p^{mm} \\ &< 8 + 4 = 12 = M^{me}, \end{aligned}$$

which again violates market clearing. Therefore,  $p^{mm} = 4$ . From above, it follows that  $q_1^{mm} = 6$  and  $q_3^{mm} \geq 6 - q_2^{mm}$ .

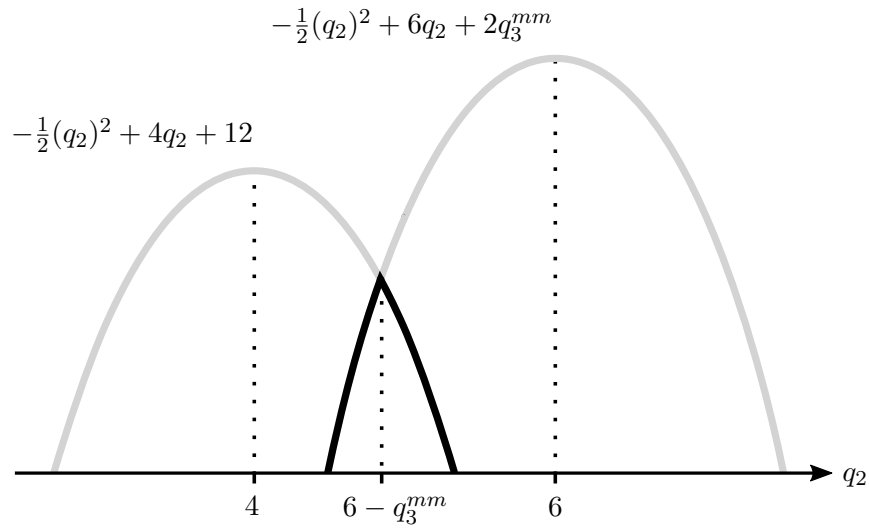
Let us now determine  $q_2^{mm}$ . For all  $q_2 \in [0, 6]$ ,  $\mu_2(q_2, \mathbf{q}_{-2}^{mm}) = q_2$  and  $\mu_3(q_2, \mathbf{q}_{-3}^{mm}) = \min\{q_3^{mm}, 6 - q_2\}$ . Thus, (16) for  $i = 2$  becomes

$$\begin{aligned} q_2^{mm} &\in \arg \max_{q_2 \in [0, 6]} \left\{ 2(q_2 + \min\{q_3^{mm}, 6 - q_2\}) - \frac{1}{2}(q_2)^2 + 4q_2 \right\} \\ &= \arg \max_{q_2 \in [0, 6]} \left\{ \min \left\{ -\frac{1}{2}(q_2)^2 + 6q_2 + 2q_3^{mm}, -\frac{1}{2}(q_2)^2 + 4q_2 + 12 \right\} \right\}. \end{aligned}$$

As illustrated in [Figure A.1](#), the solution is given by

$$q_2^{mm} = \begin{cases} 6 - q_3^{mm} & \text{if } q_3^{mm} < 2, \\ 4 & \text{if } q_3^{mm} \geq 2. \end{cases}$$

In particular, if  $q_3^{mm} = 0$ , then  $q_2^{mm} = 6$ . Recall from above that, in this case, any  $q_2^{mm} > 6$  will also yield a matching-constrained market equilibrium. Putting everything together, we get the desired characterization.



**Figure A.1:** Illustration of country 2's maximization problem in [Example 4](#)

*Note:* The figure is drawn for  $q_3^{mm} < 2$ . An increase in  $q_3^{mm}$  moves the right parabola upwards. For  $q_3^{mm} \geq 2$ , the intersection of the two parabolas is weakly to the left of 4.

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