

Harnessing beliefs to optimally disclose contestants' types

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Abstract

A contestant's effort depends on her knowledge of her rival's type. This knowledge is often limited in real-life contests. We propose a model where the principal of a contest has commitment power to verifiably disclose contestants' types. We investigate the optimal disclosure policy to stimulate contestants' efforts. Full disclosure stimulates more (less) effort than full concealment if high-types are more (less) likely than low-types. However, regardless of the likelihood of types, the optimal policy is that of contingent disclosure; it is optimal to commit to disclosing if both contestants are high types and concealing otherwise.

Keywords: Contests, Strategic Complements, Strategic Substitutes, Information.

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1 Introduction

Running relative performance contests is a popular way to stimulate workers' efforts. It is estimated that roughly between one third (Loew, 2015) and two thirds (The Wall Street Journal, 2012) of firms use some such form of internal ranking. A manager can provide some information to the competing workers about each others' *types*, and competing workers do not a priori have this information on their rivals unless informed by the manager. Workers' types can be, for instance, their past performance, their inherent ability or even their identity itself. If the manager discloses types before the proclamation of the winner of the contest (i.e., the worker who obtains the promotion, the tenure, or the bonus), then such a disclosure affects workers' efforts, and consequently the firm's profit.

Analogous information disclosure problems arise in other settings. For instance, the organizer of a sports tournament might disclose information about players that would otherwise be too costly or impossible to retrieve by the players themselves; in chess tournaments, players' Elo ranking is sometimes displayed on the pairing-list board, and in the NBA fine-tuned data collected by advanced tracking cameras are disclosed online. Similarly, scholars compete for grants by submitting research projects, and they presumably do not know who they are competing against unless the grant-awarding entity publicly discloses the list of participants. The awareness of a rival's type affects a contestant's effort. For example, an average researcher aware of being shortlisted to compete for a grant against a leading scholar might give up hope and exert little effort. In the present paper we explore the effort-maximizing disclosure policy.

Our model has an informed principal (manager, sports tournament organizer, grant-awarding entity) who maximizes the aggregate effort of two contestants (workers, players, scholars) who compete in a Tullock contest with discriminatory parameter $r \in (0, 1]$ (Tullock, 1980). First, the principal commits to a costless and verifiable disclosure policy about contestants' types (modeled as marginal costs of efforts). Then, nature assigns a type to every contestant. Contestants know their own type, but their knowledge of their *rival's* type depends on the principal's disclosure policy.¹ Our model assumes that there are two contestants. This assumption admittedly stylizes the applications we described, but it allows us to abstract from contestants' endogenous participation decisions, and thus to isolate the effect of the disclosure policy on contestants' efforts. Each contestant is of high-type with probability $p \in [0, 1]$, and low-type with the complementary probability. Types are drawn independently and p is common knowledge. We find that the probability of high-types p drives the optimal disclosure policy. We provide an intuitive explanation in what follows.

We describe two building blocks of our results. First, around the equilibrium

¹The principal commits to a disclosure policy *ex-ante*, that is, before observing types. This captures the idea that the principal chooses and commits to the *rules* of the contest, among which the disclosure policy, which are implemented every time the contest is run. This argument is similar to the one in Rayo and Segal (2010), where the precommitment power of the sender is a way to build long-run reputation in the eyes of the sequence of "short-run" receivers. In such a repeated game, losses of reputation would jeopardize the sender's long-run profits. Moreover, in Appendix B.1 we show that the principal is better off committing to her disclosure policy *ex-ante*, rather than *ex-post*, if she could choose.

of our setup the effort of the high- (low-) type and that of her rival are strategic complements (substitutes). Second, the effort of a contestant, regardless of her type, is maximum when she is up against a contestant of the same type. We call this situation an even contest, and its effort the “stimulated effort,” in contrast to an uneven contest and the “discouraged effort” where types differ.²

With these two building blocks in mind, consider the effect of the probability of high-types p on the optimal disclosure policy. Trivially, in the extreme case of $p \in \{0, 1\}$, any attempt to conceal types in order to affect efforts is fruitless since contestants know p , and thus disclosure policies are all outcome-equivalent. We analyze the case of a high (low), but non-extreme, value of p in Case I (II) below.

Case I (high p).

Consider a high-type. Under disclosure, a high-type will exert the stimulated effort with high probability (p) and the discouraged effort with low probability ($1 - p$). Under concealment, a high-type believes she is in an even (uneven) contest with high (low) probability, and knows that her most likely rival (a high-type) has the same beliefs. Thus, the resulting level of effort of the high-type under concealment is close to the effort that the principal expects from the high-type under disclosure, which in fact is the high-type’s stimulated effort with high probability (p) and the high-type’s discouraged effort with low probability ($1 - p$).

Consider a low-type. Under disclosure, a low-type will exert the stimulated effort with low probability and the discouraged effort with high probability. Under concealment, a low-type believes she is most likely up against a high-type who believes she is most likely up against another high-type, and so on. Thus, a low-type under concealment, despite knowing she is most likely up against a high-type, exerts significantly *less* effort than in a high-vs-low contest under disclosure, because she knows that her (most likely) high-type rival believes that the contest is (most likely) even, rather than uneven. This *discouragement* of a low-type is obviously only present under concealment. For brevity, we call this discouragement caused by concealment when p is high the negative p -Effect (hereafter, “ $-pE$ ”), which is stronger the more p is far from $1/2$ and close to 1.

Case II (low p).

In a nutshell, everything said for Case I carries over by just swapping high- and low-type, and changing “*less*” and “*discouragement*” into “*more*” and “*encouragement*.” In fact, the key difference is that now the p -Effect is positive. For brevity, we call this encouragement caused by concealment when p is low the positive p -Effect (hereafter, “ $+pE$ ”), which is stronger the more p is far from $1/2$ and close to 0.

The present paper makes the case that the p -Effects drive the optimal disclosure policy in contests. In fact, we find that in a standard Tullock contest aggregate effort is maximized by disclosure when $p \in (1/2, 1)$ in order to avoid the $-pE$, and by concealment when $p \in (0, 1/2)$ in order to make the most out of the $+pE$.³ Concealment and disclosure yield the same aggregate effort in the remaining cases;

²When two contestants are of the same type, they maximize the same payoff function, and this typically yields the maximum individual effort in standard two-player Tullock contests (see, e.g., Baik, 1994; Nti, 1999).

³Recall that $+pE$ and $-pE$ are present only under concealment.

namely, the symmetric prior $p = 1/2$, because $+pE$ and $-pE$ cancel out, and the degenerate prior $p \in \{0, 1\}$. These are the results of Section 4.

In Section 5, we consider a principal who can precommit to disclosing or concealing contestants' types *contingently* on the realization of types. We show that the optimal information disclosure, regardless of p , is to commit to disclosing contestants' types if contestants are both high-types and concealing all other realizations of types. Such an optimal contingent disclosure policy thus improves upon the full disclosure and full concealment policies considered in Section 4. The intuition behind this result relies again on $+pE$ and $-pE$, and it will be given in Section 5, after presenting the result.

After a discussion of the related literature in Section 2, the main body of the paper (sections 3-6) includes what is needed to understand the intuition behind the optimal disclosure policies, and everything else is moved to the appendices. In particular, Section 3 spells out the contest model and the disclosure game. Section 4 and Section 5 investigate the optimal disclosure policy for a principal who maximizes aggregate effort: in Section 4 the principal can only commit to either fully disclosing or fully concealing contestants' types, and in Section 5 the principal can commit to disclosing contestants' types contingently on the realization of types. Section 6 discusses the results. Appendix A contains the proofs and Appendix B extensions.

2 Related Literature

The most closely related literature is that on how information affects efforts in Tullock contests (e.g., Hurley and Shogren, 1998a and 1998b; Denter, Morgan and Sisak, 2011; Epstein and Mealem, 2013; Heijnen and Schoonbeek, 2016; Zhang and Zhou, 2016; Chen et al., 2017, 2018b). These papers — in contrast to ours — assume that the type of one contestant is common knowledge and that of the other is private information. In other words, the private information the principal may disclose is one-sided among the two contestants. We believe that a disclosure policy equally capable of disclosing *each* contestant's type is a sensible assumption in the applications discussed in the Introduction. From a more technical perspective, the prevailing assumption of one-sided private information contest shuts down the higher-order reasoning behind the p -Effects which drives the results of the present paper. Perhaps one reason for the popularity of the one-dimensional private information approach is its tractability, in that it admits a closed-form solution for equilibrium efforts, contrary to our context.⁴ In fact, in the words of Zhang and Zhou (2016), “It is natural to ask what happens if both contestants possess private information. Several technical challenges emerge accordingly, [...] a characterization of the equilibrium is usually not obtainable.” We sidestep the lack of a closed-form solution by showing that, in Tullock contests, equilibrium efforts always satisfy a property that depends on contestants' beliefs about the rival's type (see (6) in Appendix A), and that this property suffices to fully characterize the optimal disclosure

⁴Even if $r = 1$ in (1), a contest does not admit a closed-form solution for equilibrium efforts in our fully-private information setting unless $p \in \{0, 1/2, 1\}$. The case $p = 1/2$ is analyzed in Malueg and Yates (2004).

policy. Furthermore, this novel property happens to generalize a well-known wisdom of complete information Tullock contests — namely the ratio of types equals the ratio of efforts — to Tullock contests with a variety of contestants’ beliefs about the rival’s type.

The literature has proposed ways that sidestep the lack of a closed-form solution for the equilibrium efforts of private information Tullock contests: among others, a modified contest success function (see Wasser, 2013), a binary effort space (see Dubey, 2013), or numerical simulations (see Hurley and Shogren, 1998a; Wasser, 2013). In particular, Wasser (2013) compared full disclosure and full concealment through numerical simulations, while in Theorem 1 we formally prove his numerical result under binary distribution of types; however, not having an analytical result impeded him from explaining the reason why the probability of high- and low-types is what drives the optimal disclosure policy. The key role of the probability of high- and low-types is one of our main contributions, and we show how it extends to some non-binary distributions (see, Appendix B.3).

The present paper is, to the best of my knowledge, the first to study type-contingent information disclosure in contests with two-sided asymmetric information. Following the present paper, Chen et al. (2018a) study the joint optimal design of timing and information disclosure in Tullock contests, and Lu et al. (2018) study the optimal information disclosure in all-pay auctions, where an arbitrarily higher effort secures victory.⁵ In particular, Lu et al. (2018) extend the present analysis to all-pay auctions. Their work considerably contributes to the literature in that they provide a full characterization of the equilibrium bidding function in all-pay auctions under a regime of contingent information disclosure, where Siegel’s (2014) approach is not applicable. In contrast to our Tullock success function, the all-pay auction yields payoffs which are not quasiconcave, hence leading to a nondegenerate mixed strategy equilibrium even in the complete information case. This generates structural differences between the two settings which make the results hardly comparable. For instance, in Lu et al.’s (2018) equilibrium participation depends on beliefs and distribution of types. Furthermore, the elegant characterization of the p -Effects is possible thanks to the strategic complementarity (substitutability) of the high- (low-) type’s effort around the equilibrium, which we find in our setup with Tullock contest success function with exponent $r \leq 1$.⁶

Other branches of the literature are related to the present paper. First, the literature on the disclosure of contestants’ dynamic performance (e.g., Aoyagi, 2010; Goltsman and Mukherjee, 2011) or of the number of contestants (e.g., Myerson and Warneryd, 2006; Lim and Matros, 2009; Fu et al., 2011). Second, the literature on contestants’ incentive to acquire information (e.g., Yildirim, 2005; Denter, Morgan, and Sisak, 2011). In our model it is the principal, rather than the contestants, who has control over the information that contestants acquire. Third, the literature on contestants’ incentive to disclose their private information. While players have no incentive to do so in all-pay auctions (e.g., Kovenock, Morath and Munster, 2015),

⁵ An earlier version of the present paper is Serena (2016), which Lu et al. (2018) cite. The earliest version is Serena (2014); see the reference in Vázquez Sedano (2015).

⁶ Beside Lu et al.’s (2018) extension of our results to all-pay auctions, the comparison between expected aggregate effort under full disclosure or full concealment is in Fu, Jiao and Lu (2014) and Kovenock et al. (2015), who find that full concealment dominates full disclosure.

in Tullock contests players may want to disclose their information (Wu and Zheng, 2017). Fourth, and finally, the Bayesian persuasion literature (see Kamenica and Gentzkow, 2011, henceforth KG), where a sender commits to a disclosure policy on a stochastic state of the world before its realization. In this class of models, it is not a novelty to find optimality of contingent information disclosure, like in our setting; in particular, it is optimal for a sender (i.e., a prosecutor in KG, and the principal here) to disclose to a receiver (i.e., a judge in KG, and the set of two contestants here) only the realization which is most favorable to the sender (i.e., the defendant being guilty in KG, and both contestants being high-types here) and to pool the other signals in a strategic way (i.e., randomizing between innocent and guilty signals in KG, and concealing here). The disclosure policy space is more general under the Bayesian persuasion approach than in our setting. Nevertheless, our disclosure policy space is simpler to implement, in that it involves no stochastic messages, and thus it requires a weaker commitment condition than Bayesian persuasion. Furthermore, our disclosure policy space is not affected by the technical challenges of applying Bayesian persuasion to analyze the disclosure of *both* contestants' types. (Three such technical challenges are discussed by Zhang and Zhou, 2016; Section 4).

3 A Model of a Contest

The Contest Technology. Two contestants, indexed by $i = 1, 2$, compete for a prize of value $V > 0$ by exerting effort $e_i \geq 0$. Each contestant has a probability of winning a prize equal to⁷

$$p_i(e_i, e_j) = \begin{cases} \frac{e_i^r}{e_i^r + e_j^r} & \text{if } e_i + e_j > 0 \\ \frac{1}{2} & \text{if } e_i + e_j = 0 \end{cases} \quad (1)$$

with $i, j = 1, 2, j \neq i$, and $0 < r \leq 1$. When at least one player exerts strictly positive effort, (1) uniquely satisfies a set of appealing axioms (see Skaperdas, 1996).

The Contestants' Payoff. Contestants are risk-neutral. The cost of effort is linear, and contestant i is of type θ_i , which determines her marginal cost of effort. In particular, the payoff of a contestant of type θ_i when she exerts effort e_i and her rival exerts effort e_j is

$$p_i(e_i, e_j)V - \frac{e_i}{\theta_i}.$$

Contestant's type θ_i is an independent draw from the commonly-known prior,⁸

$$\theta_i = \begin{cases} h & \text{with probability } p \\ l & \text{with probability } 1 - p \end{cases} \quad (2)$$

with $p \in [0, 1]$ and $h > l > 0$. Thus, being a high-type rather than a low-type brings about a lower marginal cost of effort.

⁷In Appendix B.4 we briefly discuss a more general contest technology.

⁸In Appendix B.3 we discuss the case of a continuum of types.

The Timing of the Game. First, before types are realized, the principal commits to a verifiable and costless disclosure policy \mathcal{P} , which is observed by the contestants. Then, types are realized, each contestant i learns θ_i and may or may not be informed by the principal of θ_j , according to \mathcal{P} . Finally, contestants simultaneously choose efforts.

The Principal and the Disclosure Policy. The principal maximizes the expected aggregate effort when choosing the disclosure policy \mathcal{P} .⁹ In Section 4, the principal chooses between two extreme disclosure policies on types: disclosure $\mathcal{P} = \mathcal{D}$ (that is, contestants are informed of their rival’s type), and concealment $\mathcal{P} = \mathcal{C}$ (that is, contestants are not informed and contestants’ posterior belief about their rival’s types equals the prior (2)). In Section 5 we enlarge the space of \mathcal{P} ’s: the principal commits to disclosing or concealing contestants’ types *contingently* on type realizations.¹⁰ Each \mathcal{P} induces a system of beliefs for the contestants, and each system of beliefs induces an expected level of aggregate effort. We discuss the \mathcal{P} which maximizes aggregate effort in the main text, while intermediary results on how systems of beliefs affect aggregate effort appear in Appendix A.

Equilibrium. Since contestants are ex-ante symmetric, it is natural to focus on type-symmetric equilibria; that is, contestants of the same type follow the same equilibrium strategy. The existence of a unique equilibrium has already been proved in our setting (see Einy et al., 2015, and Ewerhart and Quartieri, 2019). Our results in the main text are valid for both interior and corner equilibria; formally, we assume interior equilibrium throughout the paper, and at the end of Appendix A we build on our findings to explain why considering corner equilibria does not affect our results.

4 Optimal Disclosure Policy: Disclose or Conceal?

If the principal chooses between committing to full disclosure ($\mathcal{P} = \mathcal{D}$) or full concealment ($\mathcal{P} = \mathcal{C}$) only, which one yields the greatest expected aggregate effort? When committing to a \mathcal{P} , the principal does not know whether the contest will be between two high-types, two low-types, or a high-type and a low-type. Table 1 provides a summary of the possible realizations of types, the corresponding probabilities, and the aggregate equilibrium effort (i.e., the principal’s payoff). The notation is as follows: the first subindex in e is the type of the contestant exerting effort, and the second subindex is the type of her rival, in case she is told (that is, under \mathcal{D}). Thus, for instance, e_{hl} is the equilibrium effort of a high competing against a low (under \mathcal{D}), and e_h is the equilibrium effort of a high who does not know her rival (under \mathcal{C}).

⁹In Appendix B.2 we discuss alternative objective functions for the principal.

¹⁰In Appendix B.1 we discuss the optimal disclosure policy if the principal does not have commitment power, so that the disclosure policy is *de facto* chosen ex-post.

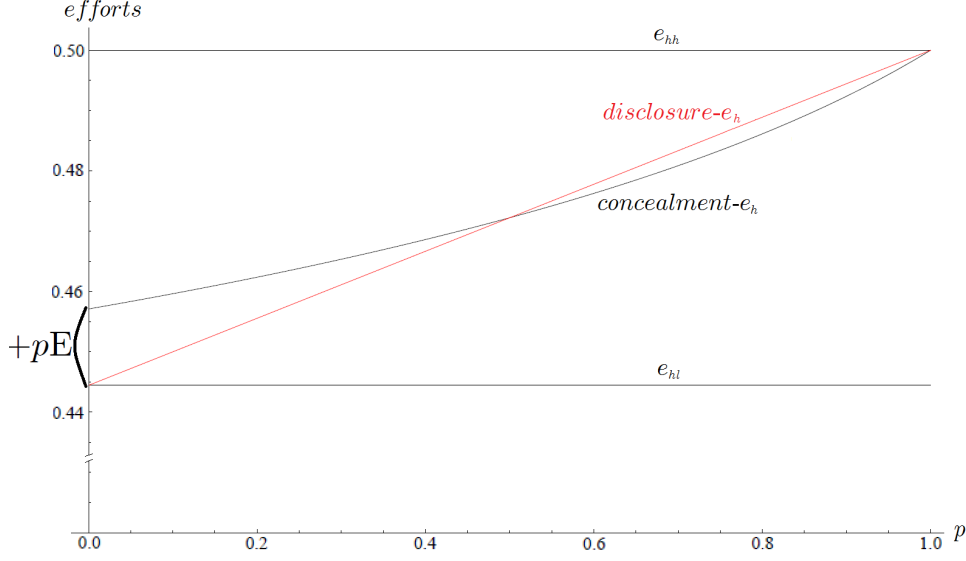


Figure 1: High-type's equilibrium efforts as functions of p assuming $r = 1$, $V = 1$, $l = 1$ and $h = 2$. The lines e_{hh} and e_{hl} are her two efforts under \mathcal{D} . The red line $disclosure-e_h$ is thus the principal's expectation of a high-type's effort under \mathcal{D} . The line $concealment-e_h$ is the effort of a high-type under \mathcal{C} . The segment called $+pE$ is the positive p -Effect, which is depicted at $p \rightarrow 0$.

Realizations of $\{\theta_1, \theta_2\}$	Prob.	Aggregate effort if \mathcal{D}	Aggregate effort if \mathcal{C}
$\{h, h\}$	p^2	$2e_{hh}$	$2e_h$
$\{l, l\}$	$(1 - p)^2$	$2e_{ll}$	$2e_l$
$\{h, l\}$ or $\{l, h\}$	$2p(1 - p)$	$e_{hl} + e_{lh}$	$e_h + e_l$

Table 1: Possible realizations of contestants' types, with corresponding probability and aggregate equilibrium effort under $\mathcal{P} = \mathcal{D}$ and $\mathcal{P} = \mathcal{C}$.

If the contest is even — $\{h, h\}$ or $\{l, l\}$ —, then \mathcal{D} maximizes aggregate effort because it prevents contestants from thinking that they are competing in an uneven contest (i.e., $2e_{hh} \geq 2e_h$ and $2e_{ll} \geq 2e_l$). If instead the contest is uneven — $\{h, l\}$ or $\{l, h\}$ —, then \mathcal{C} maximizes aggregate effort because it prevents discouragement. Therefore, finding the optimal \mathcal{P} boils down to the *ex-ante* trade-off between the benefits of \mathcal{D} if the contest is even and the benefits of \mathcal{C} if the contest is uneven. In what follows we gradually build the intuition that will eventually lead to Theorem 1 using the graphical support of Figure 1 and Figure 2.

In Figure 1 we focus on the effort of a high-type.¹¹ The horizontal lines are her two possible equilibrium efforts under disclosure — against another high-type or against a low-type —, where p does not affect efforts and $e_{hh} > e_{hl}$ because evenness of types stimulate efforts. We call $disclosure-e_h$ the effort that the principal expects from the high-type under disclosure, which equals $pe_{hh} + (1 - p)e_{hl}$. Thus, the $disclosure-e_h$ is a straight line in p going from e_{hl} to e_{hh} , as depicted. We call

¹¹Since a closed-form solution for equilibrium efforts does not exist, figures are created using numerical simulations on the system of FOCs (see (4) and (5) in Appendix A) with $r = 1$, $V = 1$, $l = 1$ and $h = 2$. This parametrization does not affect the qualitative features of those figures.

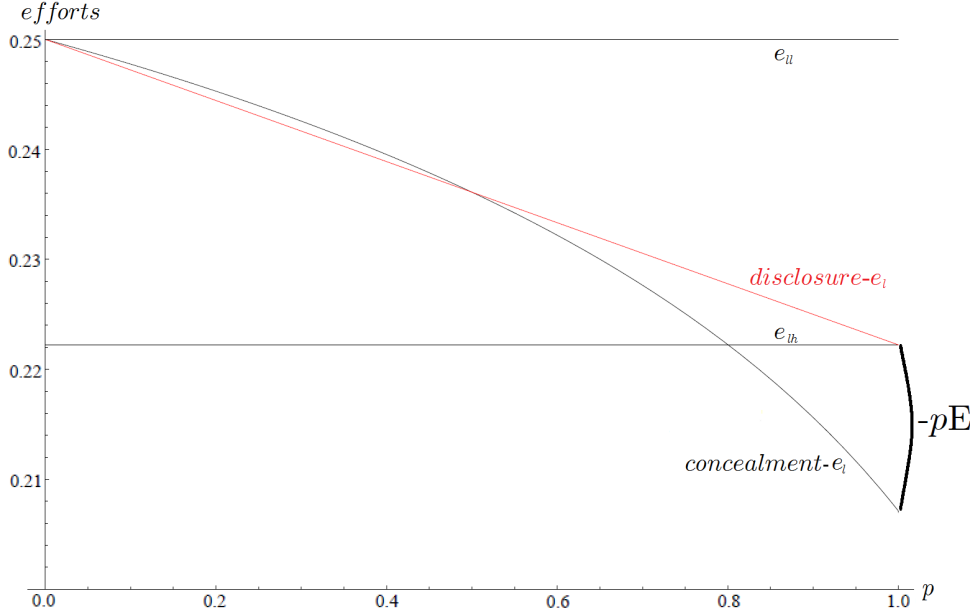


Figure 2: Low-type’s equilibrium efforts as functions of p assuming $r = 1$, $V = 1$, $l = 1$ and $h = 2$. The lines e_{ll} and e_{lh} are her two efforts under \mathcal{D} . The red line *disclosure- e_l* is thus the principal’s expectation of a low-type’s effort under \mathcal{D} . The line *concealment- e_l* is the effort of a low-type under \mathcal{C} . The segment called $-pE$ is the negative p -Effect, which is depicted at $p \rightarrow 1$.

concealment- e_h the equilibrium effort of a high-type under concealment. Ranking *concealment- e_h* and *disclosure- e_h* (and *concealment- e_l* and *disclosure- e_l* in Figure 2) unveils the optimal disclosure policy. The *concealment- e_h* is increasing in p because p increases the probability of an even contest. When $p \rightarrow 0$, the high-type believes she is up against a low-type who believes she is up against another low-type who also believes she is up against another low-type, and so on. In other words, the high-type expects an uneven contest against a rival who expects an even contest with commonly known types. This implies that *concealment- e_h* is greater than the *disclosure- e_h* (under disclosure, the contestants’ unevenness is commonly known).¹² This is what we called $+pE$. The more p increases in the interval $p \in (0, 0.5)$, the less likely there are situations where the high-type’s and the low-type’s beliefs about the evenness of the contest sharply differ (as when $p \rightarrow 0$), and this is what makes the $+pE$ gradually disappear and the *concealment- e_h* converge to the *disclosure- e_h* . When $p = 0.5$, contestants believe they are taking part in an even or uneven contest with equal probability, and they know that their rival, regardless of types, has the same beliefs: that is, the two opposing forces — namely being discouraged (by an uneven contest) and stimulated (by an even contest) — balance out, and thus the higher-order reasoning “does not distort” efforts. Additionally, when $p \in (0.5, 1)$, the *concealment- e_h* is below the *disclosure- e_h* . The reason is that a high-type believes she is more likely up against another high-type who also believes she is more likely

¹²Note that when $p = 0$, both *concealment- e_h* and *disclosure- e_h* do not matter because a high-type does not exist. Nevertheless, we start our explanation with the limiting case $p \rightarrow 0$ because this is where the p -Effect can be immediately grasped.

up against another high-type, and so on. However, the high-type knows that if she happens to be against a low-type (i.e., the less likely contingency), her low-type rival will be discouraged (as she believes she is up against a high-type who believes she is up against another high-type, and so on; shortly, the $-pE$), thus exerting low effort; the possibility of being with low probability $1 - p$ against a discouraged low-type decreases the effort of the high-type with respect to a disclosure setting, where such possibility is clearly not present.

In Figure 2 the same exercise is carried out for a low-type, with symmetric results. We briefly repeat the reasoning we made above for a high-type. When $p \rightarrow 1$, the *disclosure- e_l* is greater than the *concealment- e_l* because a low-type believes she is up against a high-type who believes she is up against another high-type, and thus the low-type exerts less effort than e_{lh} (i.e., a low-type is discouraged by $-pE$). When $p = 0.5$, the $-pE$ disappears and the *disclosure- e_l* equals the *concealment- e_l* , while when $p \in (0, 0.5)$ the *concealment- e_l* is above the *disclosure- e_l* .

Therefore, all forces affecting the high-type and low-type's efforts point at the same conclusion: for $p \in [0, 0.5]$ both *concealment- e_h* and *concealment- e_l* are greater than the disclosure ones, while for $p \in [0.5, 1]$ they are both lower. Thus, if $p \in [0, 0.5]$ concealment is the optimal policy, and if $p \in [0.5, 1]$ disclosure is the optimal policy. These results lead to the optimal \mathcal{P} of Theorem 1, which is depicted in Figure 3.¹³

Theorem 1 *Under full disclosure or full concealment only, the optimal disclosure policy for the principal is to commit to full disclosure (full concealment) if high-types are more (less) likely than low-types; that is, if $p \in [0.5, 1]$ ($p \in [0, 0.5]$). The principal is indifferent between full concealment and full disclosure if and only if the prior is degenerate, i.e., $p \in \{0, 1\}$, or symmetric, i.e., $p = 0.5$.*

Despite Theorem 1 answering a very natural question, a plausible reason why the present paper is the first to provide an answer is the well-known lack of a closed-form solution for efforts under two-sided private information Tullock contests. In the proof of Theorem 1 (and later on of Theorem 2), we sidestep the lack of a closed-form solution by showing that equilibrium efforts always satisfy a property that depends on contestants' beliefs about the rival's type (in Appendix A, see (6) for Theorem 1, and the corresponding (22)–(23) for Theorem 2), and that this property suffices to fully characterize the optimal disclosure policy.

Theorem 1 is the first of the two main results of the paper. We challenge this result with five extensions in Appendix B. First, we consider ex-post rather than ex-ante disclosure; we find that the equilibrium efforts are those of full disclosure. Second, we consider objectives other than the maximization of aggregate effort; we obtain a similar optimal disclosure policy if the principal maximizes the expected winning effort, and the opposite optimal disclosure policy to that of Theorem 1 when

¹³ Although parameters h and l do not affect the principal's optimal disclosure policy, the absolute value of the difference between aggregate effort under \mathcal{D} and under \mathcal{C} increases in the ratio h/l . In other words, the vertical scale of Figure 3 shrinks for a reduction of h/l , all the way down to the case of $h = l$, that is, a flat horizontal line depicting indifference between any \mathcal{P} for all values of p . The same holds for r and V , which do not affect the principal's optimal disclosure policy, but scale up or down efforts in each \mathcal{P} . The independence of the optimal \mathcal{P} on h, l, r and V also holds in Section 5.

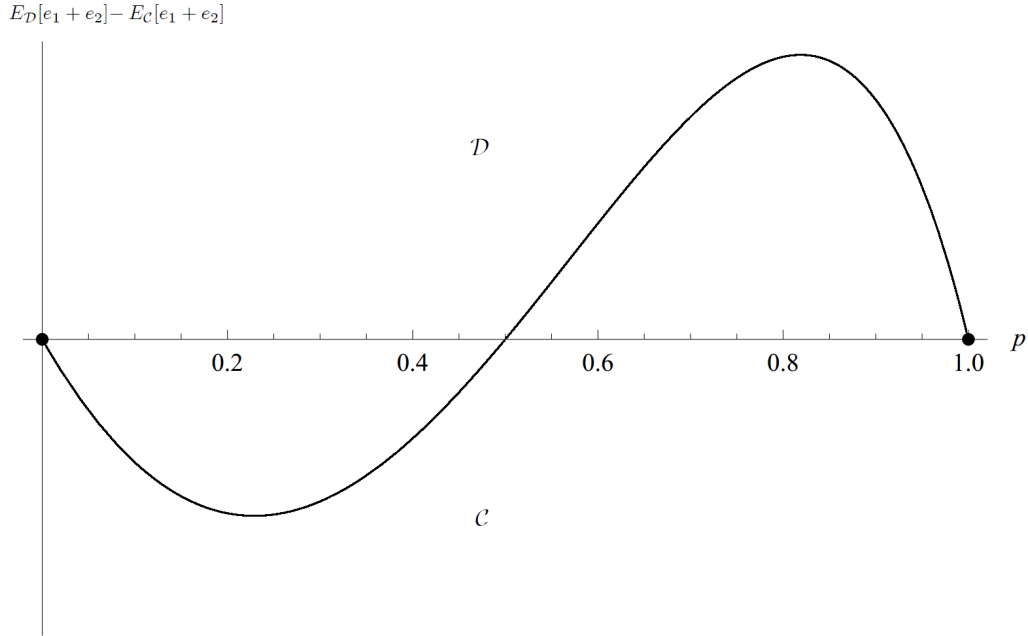


Figure 3: $E_{\mathcal{D}}[e_1 + e_2] - E_{\mathcal{C}}[e_1 + e_2]$: Difference between the expected aggregate effort under \mathcal{D} and under \mathcal{C} as a function of p , assuming $r = 1$, $V = 1$, $l = 1$ and $h = 2$.

the principal minimizes aggregate effort or maximizes the probability of a high-type winner. Third, we consider the optimal disclosure policy with two tractable cases of continuum-type distribution drawn from the literature; we find that the link between the relative likelihood of high- and low-types and optimal disclosure policy goes in the same direction as that characterized in Theorem 1. Fourth, we spell out the technical difficulties of deriving the optimal disclosure policy under a contest success function more general than (1).

5 Optimal Contingent Disclosure Policy

In Section 4, the principal could only commit to fully disclosing or fully concealing contestants' types. Can the principal do any better using a partial disclosure policy? This section shows that the answer is positive, regardless of p , in a specific family of partial disclosure policies, where the principal commits to disclosing or concealing contestants' types *contingently* on the realization of types. The principal needs a stronger commitment power in order to implement such a contingent information disclosure; online contests are a setting where such a broadening of the disclosure policy space is easily implementable.

\mathcal{P} is now a vector of three binary variables, each taking the value \mathcal{D} or \mathcal{C} , where the first (respectively, second and third) element corresponds to the disclosure choice in case of realization $\{h, h\}$ (respectively, $\{h, l\}$ and $\{l, l\}$). Realizations $\{h, l\}$ and $\{l, h\}$ are equivalent for the principal, who hence has no incentive to choose different disclosure policies to $\{h, l\}$ and $\{l, h\}$. Although the set of \mathcal{P} analyzed in this section formally nests the two extreme cases analyzed in Section 4 — $\mathcal{P} = \{\mathcal{C}, \mathcal{C}, \mathcal{C}\}$

corresponds to full concealment, and $\mathcal{P} = \{\mathcal{D}, \mathcal{D}, \mathcal{D}\}$ to full disclosure —, we keep the two sections separate because: i) the comparison of public and private information contests is of self-interest, and ii) the more sophisticated disclosure policy in this section might not be implementable in some real-life situations. As in Section 4, \mathcal{P} is publicly announced ex-ante by the principal who commits to it, and no randomization is allowed.

The result of this section is that $\mathcal{P} = \{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$ is optimal for all p 's — that is, disclosure only when both contestants are high-types yields an expected aggregate effort greater than that of full disclosure, full concealment, and any other contingent disclosure policy.

Theorem 2 *Under contingent information disclosure, the optimal disclosure policy for the principal is to commit to disclosing contestants' types only when the realization is $\{h, h\}$, if $p \in (0, 1)$. That is, $\mathcal{P} = \{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$. If $p \in \{0, 1\}$, the principal is indifferent between any disclosure policies.*

The intuition behind Theorem 2 is as follows. First, consider how $\mathcal{P} = \{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$ affects contestants' beliefs. A high-type who is not told her rival's type infers she must be up against a low-type. Thus, (A) a high-type always knows her rival's type, and (B) a low-type never knows her rival's type. More importantly, (A) and (B) are common knowledge; in particular, low-types know (A) and high-types know (B). Therefore, such a \mathcal{P} allows the principal, regardless of p , to both:

- **avoid the losses due to $-pE$.** $-pE$ is an effort loss that occurs when a low-type is up against a high-type who is not aware of being up against a low-type. This situation *does not* occur under such a \mathcal{P} because low-types know (A), and thus even if a low-type is up against a high-type, the low-type knows that the high-type is aware of being up against a low-type.

- **benefit from $+pE$.** $+pE$ is an effort gain that occurs when a high-type is up against a low-type who is not aware of being up against a high-type. This situation *does* occur under such a \mathcal{P} because high-types know (B), and thus a high-type knows that, when up against a low-type, this low-type will not be aware of being up against a high-type.

Therefore, the intuition behind Theorem 2 relies on $+pE$ and $-pE$, as the one behind Theorem 1. Yet, the optimal disclosure policy of Theorem 1 depends on p whereas the one of Theorem 2 does not. The reason is that, if only full disclosure or full concealment are possible, the principal's priority is *either* to avoid the losses due to $-pE$ (and thus disclose) when p is high *or* to make the most out of the $+pE$ (and thus conceal) when p is low, whereas if the disclosure policy space admits contingent disclosures, the principal can simultaneously achieve *both* goals for all p 's.

While the worst (or effort-minimizing) disclosure policy of Section 4 immediately follows from Theorem 1, Theorem 2 does not characterize the worst *contingent* disclosure policy. For the sake of completeness, we characterize the worst disclosure policy when contingent information disclosure is possible.

Proposition 3 *Under contingent information disclosure and $p \in (0, 1)$, the worst disclosure policy for the principal is to commit to disclosing contestants' types only when the realization is $\{l, l\}$. That is, $\mathcal{P} = \{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$.*

The intuition behind Proposition 3 is the mirror image of that of Theorem 2; namely by disclosing only realization $\{l, l\}$ the principal suffers the losses due to $-pE$ and at the same time she does not obtain the benefits of $+pE$. Additionally, Proposition 3, together with Theorem 2 and Theorem 1, yields the full ranking of contingent disclosure policies;

Corollary 4 *Consider contingent information disclosure and $p \in (0, 1)$. If $p \in (0, 0.5)$, the full ranking of policies is*

$$\{\mathcal{D}, \mathcal{C}, \mathcal{C}\} \succ \{\mathcal{C}, \mathcal{C}, \mathcal{C}\} \succ \{\mathcal{D}, \mathcal{D}, \mathcal{D}\} \succ \{\mathcal{C}, \mathcal{C}, \mathcal{D}\}.$$

If $p \in (0.5, 1)$, the full ranking of policies is

$$\{\mathcal{D}, \mathcal{C}, \mathcal{C}\} \succ \{\mathcal{D}, \mathcal{D}, \mathcal{D}\} \succ \{\mathcal{C}, \mathcal{C}, \mathcal{C}\} \succ \{\mathcal{C}, \mathcal{C}, \mathcal{D}\}.$$

If $p = 0.5$, the full ranking of policies is

$$\{\mathcal{D}, \mathcal{C}, \mathcal{C}\} \succ \{\mathcal{D}, \mathcal{D}, \mathcal{D}\} \sim \{\mathcal{C}, \mathcal{C}, \mathcal{C}\} \succ \{\mathcal{C}, \mathcal{C}, \mathcal{D}\}.$$

6 Discussion

We have studied a model of contest where contestants are privately informed of their types and the principal can commit to publicly disclosing contestants' types. We have characterized the optimal disclosure policy when the principal maximizes the expected aggregate effort of the contestants. In particular, full disclosure extracts more (less) expected aggregate effort than full concealment if high-types are more (less) likely than low-types. Moreover, if contingent information disclosure is possible, the optimal disclosure policy is to commit to disclosing contestants' types only when the realization of types which is most favorable for the principal — i.e., contestants are all high-types — and to concealing the rest.

We have shown that the effect of the disclosure policy on the contestants' higher-order reasoning (which we called $+pE$ and $-pE$) is of first-order importance. In fact, we find that $+pE$ and $-pE$ are the only forces that affect the efforts of the high-type and low-type with opposite sign when high-types are more and less likely than low-types. This asymmetry is the by-product of the fact that, around the equilibrium, a high-type's strategy shows strategic complementarity and a low-type's strategy shows strategic substitutability. By no means do we attempt to make a statement that optimal disclosure policies depend in general only on the p -Effects. Rather, two-player Tullock contests are a suitable environment for singling out and stressing the relevance of the p -Effects which, although not convoluted, have not received attention in earlier papers, to the best of our knowledge.

Three shortcomings of our analysis pinpoint room for future research. First, we ignored contestants' selection. It may be effort-improving to endow the model with an entry fee capable of sifting out weak applicants. Second, we abstracted away from contestants' participation decision. The announced disclosure policy itself may affect contestants' willingness to participate, and we conjecture that concealment deters

the low-types from entering, and that this might be beneficial in terms of expected aggregate effort. Third, the disclosure of information about other contestants that would otherwise be unknown to contestants has other consequences in addition to the one considered in the present paper. In fact, disclosure may also trigger communication among contestants, affect the external visibility of contestants, and create animosity among contestants. We abstracted away from these effects and focused here on the direct effects on efforts of making contestants' types publicly available to other contestants.

Appendix A. Proofs.

In this Appendix, we prove Theorem 1 and Theorem 2 (together with the full ranking of policies of Proposition 3 and Corollary 4) by first deriving some equilibrium properties as “**Preliminaries**”, and second building on those preliminaries to prove the main results in a number of “**Steps**”.

PROOF OF THEOREM 1

We adopt the compact notation $\pi^{\mathcal{D}-\mathcal{C}}$ for $E_{\mathcal{D}}[e_1 + e_2] - E_{\mathcal{C}}[e_1 + e_2]$; that is, the difference in the expected sum of efforts under \mathcal{D} and under \mathcal{C} , which is depicted in Figure 3 as a function of p .

First, we spell out some **Preliminaries**. Second, we prove Theorem 1 in three steps. **Step 1:** $\pi^{\mathcal{D}-\mathcal{C}} = 0$ iff $p \in \{0, 0.5, 1\}$. **Step 2:** The derivative of $\pi^{\mathcal{D}-\mathcal{C}}$ with respect to p at $p = 0.5$ is strictly positive. **Step 3:** $\pi^{\mathcal{D}-\mathcal{C}}$ is continuous in p . These three results together lead to the sign of $\pi^{\mathcal{D}-\mathcal{C}}$ as in Figure 3, hence characterizing the optimal disclosure policy of Theorem 1 without the need for closed-form equilibrium efforts under \mathcal{C} .

Preliminaries. The existence and uniqueness of equilibrium in pure strategies is already present in the literature.¹⁴

Under \mathcal{D} , as known in the literature (e.g., Nti, 1999), equilibrium efforts are¹⁵

$$e_{hh} = \frac{rh}{4}V, \quad e_{hl} = \frac{rh^{r+1}l^r}{(h^r + l^r)^2}V, \quad e_{lh} = \frac{rl^{r+1}h^r}{(h^r + l^r)^2}V, \quad e_{ll} = \frac{rl}{4}V. \quad (3)$$

Under \mathcal{C} , the system of FOCs has two unknowns (e_h and e_l), and reads

$$\text{(FOC of } h\text{-type) :} \quad p \frac{r}{4e_h}V + (1-p) \frac{re_h^{r-1}e_l^r}{[e_h^r + e_l^r]^2}V = \frac{1}{h}, \quad (4)$$

$$\text{(FOC of } l\text{-type) :} \quad (1-p) \frac{r}{4e_l}V + p \frac{re_l^{r-1}e_h^r}{[e_h^r + e_l^r]^2}V = \frac{1}{l}. \quad (5)$$

The FOCs are necessary and sufficient to characterize the best reply, which is continuously differentiable and bounded.¹⁶ To lighten the notation we omit the dependencies of e_h and e_l on p, h, l, r and V . A type-symmetric equilibrium is a pair (e_h, e_l) satisfying (4) and (5).

¹⁴See Szidarovszky and Okuguchi (1997) for the existence and uniqueness under complete information. Under incomplete information, Einy et al. (2015) and Ewerhart and Quartieri (2019) prove existence and uniqueness in a setting that nests our model.

¹⁵The notation for the subindexes of equilibrium efforts is the one introduced at the beginning of Section 4.

¹⁶See Yildirim (2005) and Morgan and Várdy (2007).

Finally, isolate the second addends of the left-hand sides in (4) and (5), consider their ratio,

$$\begin{aligned} \frac{(1-p)re_h^{r-1}e_l^r}{pre_l^{r-1}e_h^r} &= \frac{\frac{1}{h} - p\frac{r}{4e_h}V}{\frac{1}{l} - (1-p)\frac{r}{4e_l}V} \\ \iff \frac{(1-p)e_l}{pe_h} &= \frac{\frac{1}{h} - p\frac{r}{4e_h}V}{\frac{1}{l} - (1-p)\frac{r}{4e_l}V} \end{aligned}$$

cross-multiply,

$$\begin{aligned} \frac{(1-p)e_l}{l} - (1-p)^2\frac{r}{4}V &= \frac{pe_h}{h} - p^2\frac{r}{4}V \\ \iff \frac{4}{rV}\frac{(1-p)e_l}{l} &= \frac{4}{rV}\frac{pe_h}{h} + (1-2p), \end{aligned}$$

and finally use the expressions of e_{hh} and e_{ll} in (3), in order to derive the following key relation between equilibrium efforts under \mathcal{D} and under \mathcal{C} ,

$$\frac{e_l}{e_{ll}}(1-p) = \frac{e_h}{e_{hh}}p + (1-2p). \quad (6)$$

Using the above preliminaries (i.e., (3)–(6)), we are ready to proceed with the three main steps needed to prove Theorem 1.

Step 1. We show that $\pi^{\mathcal{D}-\mathcal{C}} = 0$ iff $p \in \{0, 0.5, 1\}$. First, we analyze when $\pi^{\mathcal{D}-\mathcal{C}}$ takes the value 0.

$$\pi^{\mathcal{D}-\mathcal{C}} = p^2[2e_{hh} - 2e_h] + 2p(1-p)[e_{hl} + e_{lh} - e_h - e_l] + (1-p)^2[2e_{ll} - 2e_l] = 0,$$

i.e.,

$$p^2e_{hh} + (1-p)^2e_{ll} + p(1-p)[e_{hl} + e_{lh}] - pe_h - (1-p)e_l = 0. \quad (7)$$

If $p = 0$ or $p = 1$, by $e_l = e_{ll}$ and $e_h = e_{hh}$ respectively, (7) holds and $\pi^{\mathcal{D}-\mathcal{C}} = 0$. Thus, from now on we can focus on $p \in (0, 1)$.

Substitution of (3) and (6) into (7) yields

$$\begin{aligned} p^2\frac{rh}{4}V + (1-p)^2\frac{rl}{4}V + p(1-p)\frac{r(h+l)h^rl^r}{(h^r+l^r)^2}V - p\frac{h+l}{h}e_h - \frac{1-2p}{4}rV &= 0, \\ pr\frac{h+l}{4}V + (1-p)\frac{r(h+l)h^rl^r}{(h^r+l^r)^2}V &= \frac{h+l}{h}e_h, \\ pe_{hh} + (1-p)e_{ll} &= e_h. \end{aligned} \quad (8)$$

Hence, (8) is a condition for the indifference between \mathcal{D} and \mathcal{C} written in terms of the efforts exerted by the high-type only. This condition coincides with equality of *concealment- e_h* and *disclosure- e_h* (see Figure 1). With a similar procedure used to find (8) – that is, by substituting (6) into e_h rather than into e_l – we can obtain the value of e_l for which the administrator is indifferent between \mathcal{D} and \mathcal{C} , which symmetrically to (8) is

$$(1-p)e_{ll} + pe_{lh} = e_l. \quad (9)$$

This condition coincides with equality of *concealment*- e_l and *disclosure*- e_l (see Figure 2). We plug (8) and (9) into (4) and see if any $p \in (0, 1)$ solves the resulting equation – thus, yielding $\pi^{\mathcal{D}-\mathcal{C}} = 0$.¹⁷

First, use (3) to rewrite the indifference conditions (8) and (9) for e_l and e_h as

$$e_h = rh \frac{p(h^r + l^r)^2 + 4(1-p)h^r l^r}{4(h^r + l^r)^2} V, \quad (10)$$

$$e_l = rl \frac{(1-p)(h^r + l^r)^2 + 4ph^r l^r}{4(h^r + l^r)^2} V. \quad (11)$$

These efforts are those that, if exerted under full concealment, lead to indifference between \mathcal{C} and \mathcal{D} . Now, using (4) and (5), we check whether these effort levels are reached for some parameter values. Hence, we rewrite (4) as

$$pr \frac{h}{4} V + (1-p)r \frac{he_h^r e_l^r}{(e_h^r + e_l^r)^2} V = e_h. \quad (12)$$

Plugging (10) into the right-hand side of (12), and after simple simplifications, we obtain the following

$$\frac{e_h^r e_l^r}{(e_h^r + e_l^r)^2} = \frac{h^r l^r}{(h^r + l^r)^2}. \quad (13)$$

Finally, we plug (10) and (11) where we defined $J = p(h^r + l^r)^2 + 4(1-p)h^r l^r$ and $K = (1-p)(h^r + l^r)^2 + 4ph^r l^r$ into (13), and obtain

$$\begin{aligned} \frac{h^r l^r J^r K^r}{(h^r J^r + l^r K^r)^2} &= \frac{h^r l^r}{(h^r + l^r)^2} \\ \iff l^{2r} K^r (J^r - K^r) &= h^{2r} J^r (J^r - K^r), \end{aligned} \quad (14)$$

and the unique solution of (14) is $J = K$, which is equivalent to

$$\begin{aligned} p(h^r + l^r)^2 + 4(1-p)h^r l^r &= (1-p)(h^r + l^r)^2 + 4ph^r l^r \\ \iff 4(1-2p)h^r l^r &= (1-2p)(h^r + l^r)^2, \end{aligned}$$

whose unique solution is $p = 0.5$. Similar algebra shows that (10), (11) and $p = 0.5$ satisfy (5) – besides satisfying (4) as proved. Hence, we proved that there are only three values of p for which $\pi^{\mathcal{D}-\mathcal{C}} = 0$: 0, 0.5, and 1.

Step 2. We write the system (4) and (5) as a unique equation in terms of e_h and parameters only, and then we make use of the implicit function theorem to evaluate the derivative of $\pi^{\mathcal{D}-\mathcal{C}}$ in $p = 0.5$, and prove that it is strictly positive. That is,

$$\left. \frac{\partial \pi^{\mathcal{D}-\mathcal{C}}}{\partial p} \right|_{p=0.5} > 0.$$

¹⁷Remark: the fact that (8) and (9) are sufficient for $\pi^{\mathcal{D}-\mathcal{C}} = 0$ could have already been noticed in (7), but we needed to use (6) to show that they are also necessary for $\pi^{\mathcal{D}-\mathcal{C}} = 0$.

Remember that efforts under \mathcal{D} are not functions of p , unlike the efforts under \mathcal{C} . We omitted this detail so far in the notation, and we now write it when it would otherwise yield confusion as we need to differentiate with respect to p . To simplify $\pi^{\mathcal{D}-\mathcal{C}}$ we use the same steps used to move from (7) to (8), where we simplified a p , and get

$$\begin{aligned} \left. \frac{\partial}{\partial p} [p^2 e_{hh} + p(1-p)e_{hl} - p e_h(p)] \right|_{p=0.5} &> 0, \\ \left[2p e_{hh} + e_{hl} - 2p e_{hl} - e_h(p) - p \frac{\partial e_h(p)}{\partial p} \right] \bigg|_{p=0.5} &> 0, \\ e_{hh} - e_h(0.5) &> \frac{1}{2} \left(\left. \frac{\partial e_h(p)}{\partial p} \right|_{p=0.5} \right). \end{aligned} \quad (15)$$

When $p = 0.5$, we know from Step 1 that $\pi^{\mathcal{D}-\mathcal{C}} = 0$, and hence from (8), $e_h(0.5) = \frac{e_{hh} + e_{hl}}{2}$. Therefore, (15) is equivalent to

$$e_{hh} - e_{hl} > \left. \frac{\partial e_h(p)}{\partial p} \right|_{p=0.5} \quad (16)$$

The left-hand side of (16) is known by (3). The right-hand side is trickier. First, isolate e_l in (6) and use e_{hh} and e_{ll} from (3) to obtain:

$$e_l = \frac{4p l e_h + (1-2p)r l h V}{4(1-p)h}. \quad (17)$$

Use (17) into (4), and obtain

$$f(e_h, p) \equiv p \frac{r}{4e_h} V + 4^r h^r l^r r \frac{(1-p)^{r+1} e_h^{r-1} [h(1-2p)V + 4p e_h]^r}{[4^r h^r (1-p)^r e_h^r + (4p l e_h + h l (1-2p)V)^r]^2} - \frac{1}{h} = 0.$$

The defined $f(e_h, p)$ is an equation in p and e_h only, and hence by the implicit function theorem

$$\left. \frac{\partial e_h(p)}{\partial p} \right|_{p=0.5} = - \frac{\left. \frac{\partial f(e_h, p)}{\partial p} \right|_{p=0.5}}{\left. \frac{\partial f(e_h, p)}{\partial e_h} \right|_{p=0.5}}. \quad (18)$$

We will eventually plug (18) into (16) to conclude the proof of Step 2. We start with the denominator of (18):

$$\begin{aligned} \left. \frac{\partial f(e_h, p)}{\partial e_h} \right|_{p=0.5} &= \frac{\partial f(e_h, 0.5)}{\partial e_h} \\ &= \frac{\partial}{\partial e_h} \left[\frac{r}{8e_h} V + \frac{h^r l^r r}{2(h^r + l^r)^2 e_h} V \right] \bigg|_{p=0.5} \\ &= -\frac{r}{e_h^2} \left[\frac{h^{2r} + l^{2r} + 6h^r l^r}{8(h^r + l^r)^2} V \right] \bigg|_{p=0.5} \end{aligned} \quad (19)$$

Note that when $p = 0.5$ equilibrium effort is $e_h = rh \frac{h^{2r} + l^{2r} + 6h^r l^r}{8(h^r + l^r)^2} V$. This is easy to verify, since $p = 0.5$ corresponds to the contest of Malueg and Yates (2004) — see their expression (26). We use this expression into (19) and obtain

$$\left. \frac{\partial f(e_h, p)}{\partial e_h} \right|_{p=0.5} = - \left. \frac{1}{h e_h} \right|_{p=0.5}.$$

Hence, expression (18) reads

$$\begin{aligned} \left. \frac{\partial e_h(p)}{\partial p} \right|_{p=0.5} &= - \frac{\left. \frac{\partial f(e_h, p)}{\partial p} \right|_{p=0.5}}{\left. \frac{\partial f(e_h, p)}{\partial e_h} \right|_{p=0.5}} \\ &= \left(h e_h \left. \frac{\partial f(e_h, p)}{\partial p} \right|_{p=0.5} \right) \\ &= \frac{rh}{4} V + 4^r h^{r+1} l^r r e_h^r \left. \frac{\partial}{\partial p} \left[\frac{(1-p)^{r+1} [h(1-2p)V + 4pe_h]^r}{[4^r h^r (1-p)^r e_h^r + (4ple_h + hl(1-2p)V)^r]^2} \right] \right|_{p=0.5} \\ &= \frac{rh}{4} V + 4^r h^{r+1} l^r r e_h^r \left. \frac{\partial}{\partial p} \left[\frac{a(p)b(p)}{[c(p)]^2} \right] \right|_{p=0.5} \end{aligned} \quad (20)$$

where we defined

$$\begin{aligned} a(p) &= (1-p)^{r+1}, \\ b(p) &= [h(1-2p)V + 4pe_h]^r, \\ c(p) &= 4^r h^r (1-p)^r e_h^r + (4ple_h + hl(1-2p)V)^r. \end{aligned}$$

Hence,

$$\left. \frac{\partial}{\partial p} \left[\frac{a(p)b(p)}{[c(p)]^2} \right] \right|_{p=0.5} = \left. \frac{a'(p)b(p) + a(p)b'(p)}{[c(p)]^2} \right|_{p=0.5} - \frac{2a(p)b(p)c'(p)}{[c(p)]^3} \Big|_{p=0.5}. \quad (21)$$

From the definitions of the functions a , b and c compute their values and their derivatives when $p = 0.5$.

$$\begin{aligned} a(0.5) &= \frac{1}{2^{r+1}} & a'(0.5) &= -\frac{r+1}{2^r} \\ b(0.5) &= 2^r e_h^r & b'(0.5) &= 2^{r+1} r e_h^{r-1} [e_h - hV/2] \\ c(0.5) &= 2^r e_h^r (h^r + l^r) & c'(0.5) &= -2^{r+1} h^r r e_h^r + 2^{r+1} l^r r e_h^{r-1} (e_h - hV/2) \end{aligned}$$

Plug these results into (21) to write (20) in the following way¹⁸

$$\begin{aligned} \left. \frac{\partial e_h(p)}{\partial p} \right|_{p=0.5} &= \frac{rh}{4} V + \\ &\quad -4^r h^{r+1} l^r r e_h^r \left[\frac{e_h^r (r+1) - r e_h^{r-1} (e_h - hV/2)}{[2^r e_h^r (h^r + l^r)]^2} + \frac{r e_h^r 2^{r+1} [-h^r e_h^r + l^r e_h^{r-1} (e_h - hV/2)]}{[2^r e_h^r (h^r + l^r)]^3} \right] \\ &= \frac{rh}{4} V + 4h^{r+1} l^r r \frac{l^r - h^r}{(h^r + l^r) [(h^r + l^r)^2 + 4h^r l^r]} V - h^{r+1} l^r r \frac{l^r (2r+1) + h^r (1-2r)}{(h^r + l^r)^3} V, \end{aligned}$$

¹⁸For the sake of brevity, we use e_h rather than $e_h|_{p=\frac{1}{2}}$.

where we used the fact that $e_h = rh \frac{h^{2r} + l^{2r} + 6h^r l^r}{8(h^r + l^r)^2} V$ when $p = 0.5$. Therefore, we can finally evaluate expression (16).

$$\begin{aligned}
\frac{rh}{4} - rh \frac{h^r l^r}{(h^r + l^r)^2} &> \frac{rh}{4} + 4h^{r+1} l^r r \frac{l^r - h^r}{(h^r + l^r) [(h^r + l^r)^2 + 4h^r l^r]} + \\
&\quad - h^{r+1} l^r r \frac{l^r(2r+1) + h^r(1-2r)}{(h^r + l^r)^3} \\
&\iff 2r \frac{l^r - h^r}{(h^r + l^r)^2} > 4 \frac{l^r - h^r}{[(h^r + l^r)^2 + 4h^r l^r]} \\
&\iff 2(h^r + l^r)^2 > r [(h^r + l^r)^2 + 4h^r l^r].
\end{aligned}$$

By $r \leq 1$, it suffices to show that

$$\begin{aligned}
2(h^r + l^r)^2 &> (h^r + l^r)^2 + 4h^r l^r \\
&\iff (h^r - l^r)^2 > 0,
\end{aligned}$$

and the result follows.

Step 3. The continuity of e_h and e_l in p directly follows from the Maximum Theorem applied to contestants' payoff, which is continuous and strictly concave in own effort. The continuity of $\pi^{\mathcal{D}-\mathcal{C}}$ in p follows from the continuity of e_h and e_l .

PROOF OF THEOREM 2 AND PROPOSITION 3

If $p \in \{0, 1\}$, the principal is trivially indifferent between all disclosure policies. The ranking between $\pi^{\{\mathcal{D}, \mathcal{D}, \mathcal{D}\}}$ and $\pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}}$ is known from Theorem 1. Hence, in what follows, we focus on $p \in (0, 1)$ and on all rankings except the one between $\pi^{\{\mathcal{D}, \mathcal{D}, \mathcal{D}\}}$ and $\pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}}$.

First, we spell out some **Preliminaries**, including in Lemma 5 two key monotonicities of efforts in beliefs. Second, building on Preliminaries and Lemma 5, we prove Theorem 2 in three steps. Denoting by $\pi^{\mathcal{P}}$ the expected sum of efforts under disclosure policy \mathcal{P} , we prove that $\pi^{\{\mathcal{D}, \mathcal{D}, \mathcal{D}\}} > \pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}}$ (**Step 1**), that $\pi^{\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}} > \pi^{\{\mathcal{D}, \mathcal{D}, \mathcal{D}\}}$ (**Step 2**), and that $\pi^{\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}} > \pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}}$ (**Step 3**). Finally, note that in all the remaining policies (i.e., $\{\mathcal{C}, \mathcal{D}, \mathcal{D}\}$, $\{\mathcal{D}, \mathcal{C}, \mathcal{D}\}$, $\{\mathcal{D}, \mathcal{D}, \mathcal{C}\}$, $\{\mathcal{C}, \mathcal{D}, \mathcal{C}\}$) contestants perfectly infer types, and thus they are outcome-equivalent to $\mathcal{P} = \{\mathcal{D}, \mathcal{D}, \mathcal{D}\}$. Theorem 2 thus follows from the three steps.

Finally, in order to prove Proposition 3, notice the **Steps 1-2** imply $\pi^{\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}} > \pi^{\{\mathcal{D}, \mathcal{D}, \mathcal{D}\}} > \pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}}$, and **Step 3** implies that $\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}$ is not the best policy. In the additional **Step 4** below we prove that $\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}$ is not the worst policy either. By Theorem 1 the full ranking of Corollary 4 then follows.

Preliminaries. First, we derive two key equilibrium properties, (22) and (23), which hold for the two new disclosure policies, $\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$ and $\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$, by plugging the appropriate p .

In particular, for policy $\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$ we plug $p = 1$ in (5), as a low-type observing \mathcal{C} under policy $\mathcal{P} = \{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$ is sure she is up against a high-type, and following the same steps used to derive (6) from (4) and (5) we obtain

$$4le_h - 4(1-p)he_l = prhlV, \quad (22)$$

and similarly, for policy $\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$, we plug $p = 0$ in (4), as a high-type observing \mathcal{C} under policy $\mathcal{P} = \{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$ is sure she is up against a low-type, and following the same steps used to derive (6) from (4) and (5) we obtain

$$4he_l - 4lpe_h = (1-p)rhlV. \quad (23)$$

The second preliminary result is the following key lemma.

Lemma 5 *The equilibrium effort of the low-type decreases in the high-type's belief of being in an even contest.*

The equilibrium effort of the high-type increases in the low-type's belief of being in an even contest.

Proof of Lemma 5. For the sake of the proof of this lemma only, we denote by p_h (p_l) the belief of a high (low) type of being in an even contest; that is, of being against another high (low) type. Hence, we rewrite the system of FOCs (4) and (5) as

$$\begin{cases} p_h A + (1-p_h)B = \frac{1}{h} \\ p_l C + (1-p_l)D = \frac{1}{l} \end{cases}, \quad (24)$$

where we define

$$A \equiv \frac{r}{4e_h}V, \quad B \equiv \frac{re_h^{r-1}e_l^r}{[e_h^r + e_l^r]^2}V, \quad C \equiv \frac{r}{4e_l}V, \quad D \equiv \frac{re_l^{r-1}e_h^r}{[e_h^r + e_l^r]^2}V.$$

We prove the statements of the lemma building on five intermediary results.

First, we show that

$$e_h > e_l. \quad (25)$$

Assume by contradiction that $e_h \leq e_l$. Since $\frac{1}{h} < \frac{1}{l}$, (24) implies

$$p_h A + (1-p_h)B < p_l C + (1-p_l)D. \quad (26)$$

Also, it is routine to show that $A \geq B$, $A \geq C$, $C \geq D$ and $B \geq D$. Thus, $A \geq \max\{B, C\} \geq \min\{B, C\} \geq D$. If $B \geq C$, then a contradiction is immediately reached by $A \geq B \geq C \geq D$ and (26). If instead $C > B$, then using the definitions of B and C and cross-multiplying, we obtain $[e_h^r + e_l^r]^2 > 4e_l^{r+1}e_h^{r-1}$, which requires $[e_h^r + e_l^r]^2 > 4e_l^{2r}$, which in turn can be written as $[e_h^r + 3e_l^r][e_h^r - e_l^r] > 0$. This contradicts $e_h \leq e_l$.

Second, we show that

$$\frac{\partial [p_h A + (1-p_h)B]}{\partial e_h} \leq 0 \leq \frac{\partial [p_h A + (1-p_h)B]}{\partial e_l}. \quad (27)$$

Result (27) has an easy economic interpretation; a high type's marginal payoff from higher effort is decreasing in e_h (as the payoff function is concave), and is increasing in e_l for $e_l < e_h$ (encouragement effect). In fact, A decreases in e_h . B decreases in e_h because its numerator decreases in e_h and its denominator increases in e_h . A is constant in e_l . B increases in e_l because

$$\begin{aligned}\frac{\partial B}{\partial e_l} &= r e_h^{r-1} \frac{r e_l^{r-1} [e_h^r + e_l^r] - 2r e_l^{2r-1}}{[e_h^r + e_l^r]^3} V \\ &= r^2 e_h^{r-1} e_l^{r-1} \frac{e_h^r - e_l^r}{[e_h^r + e_l^r]^3} V > 0,\end{aligned}$$

where the last inequality follows from (25).

Third, as can be proved with similar, thus omitted, steps as for (27),

$$\frac{\partial [p_l C + (1 - p_l) D]}{\partial e_h} \leq 0, \quad \frac{\partial [p_l C + (1 - p_l) D]}{\partial e_l} \leq 0. \quad (28)$$

Result (28) has an analogous economic interpretation as the one provided for (27).

Fourth, we show that efforts are smaller than their complete-information levels in an even contest, or formally,

$$e_h \leq \frac{rhV}{4} \quad \text{and} \quad e_l \leq \frac{rlV}{4}. \quad (29)$$

Consider the first equation of (24). If $p_h = 1$, $e_h = rhV/4$. Consider now the effect of lowering $p_h (< 1)$. Then, by $A \geq B$, the convex combination $p_h A + (1 - p_h) B$ decreases in p_h . To keep it equal to the constant $1/h$, then e_h must decrease because both A and B are decreasing functions of e_h , as proved above. Hence, $e_h \leq rhV/4$. The proof of $e_l \leq rlV/4$ is analogous.

Fifth, following the same steps used to derive (6), but for the general beliefs p_h and p_l as defined in this proof, we generalize (6) to

$$\frac{\frac{4e_l}{rlV} - p_l}{1 - p_l} = \frac{\frac{4e_h}{rhV} - p_h}{1 - p_h}, \quad (30)$$

and recall that both e_l and e_h depend on (p_h, p_l) .

The *first* statement of Lemma 5 can be written as $\partial e_l / \partial p_h < 0$. Assume by contradiction that $\partial e_l / \partial p_h \geq 0$. The left-hand side of (30) increases in e_l , and thus by $\partial e_l / \partial p_h \geq 0$ it also increases in p_h . Since the left-hand side of (30) increases in p_h , also the right-hand side of (30) has to increase in p_h . However, $\frac{x - p_h}{1 - p_h}$ decreases in p_h whenever $x \leq 1$ (which holds by (29)), hence the only way to have the right-hand side of (30) increasing in p_h is that $\partial e_h / \partial p_h \geq 0$. By (28), $\partial e_l / \partial p_h \geq 0$ and $\partial e_h / \partial p_h \geq 0$ lead to a contradiction. Therefore, $\partial e_l / \partial p_h < 0$, proving the first statement of Lemma 5.

The *second* statement of Lemma 5 can be written as $\partial e_h / \partial p_l > 0$. First, as we just proved, $\partial e_l / \partial p_h < 0$, and hence by (28) and $p_l C + (1 - p_l) D = 1/l$, it must be that $\partial e_h / \partial p_h > 0$. Second, by (27) and $p_h A + (1 - p_h) B = 1/h$ we obtain $\partial e_h / \partial p_l > 0 \iff \partial e_l / \partial p_l > 0$. Therefore, the last step to finish the proof of the second statement of Lemma 5 is to assume that $\partial e_h / \partial p_l \leq 0$ and $\partial e_l / \partial p_l \leq 0$

and obtain a contradiction. For brevity, we denote these two conditions $e'_l \leq 0$ and $e'_h \leq 0$ in what follows. We differentiate the second equation of (24) with respect to p_l (for the differential of $(1 - p_h)B$ we apply the formula in (21))

$$\begin{aligned} & \frac{\overbrace{[re_l^{r-1} + p_l r (r-1) e_l^{r-2} e'_l]}^E \overbrace{e_l^r - p_l r^2 e_l^{2r-2} e'_l]^r}^F}{4e_l^{2r}} V + \\ & + \frac{\overbrace{-re_l^{r-1} e_h^r + (1-p_l) [r(r-1) e_l^{r-2} e_h^r e'_l + re_l^{r-1} re_h^{r-1} e'_h]}^G}{[e_h^r + e_l^r]^2} V + \\ & - \frac{\overbrace{2(1-p_l) re_l^{r-1} e_h^r [re_h^{r-1} e'_h + re_l^{r-1} e'_l]}^I}{[e_h^r + e_l^r]^3} V = 0. \end{aligned}$$

We prove that $E + F + G + H + I > 0$ in order to achieve a contradiction and thus end the proof. Term F is trivially positive.¹⁹ Term $E + G$ is positive since it can be written as

$$E + G = \frac{re_l^{2r-1} [e_h^r - e_l^r]^2 + p_l r (r-1) e_l^{2r-2} e'_l [e_h^r + e_l^r]^2}{4e_l^{2r} [e_h^r + e_l^r]^2} > 0.$$

And similarly,

$$\begin{aligned} H + I &= (1-p_l) \frac{[r(r-1) e_l^{r-2} e_h^r e'_l + re_l^{r-1} re_h^{r-1} e'_h] [e_h^r + e_l^r] - 2re_l^{r-1} e_h^r [re_h^{r-1} e'_h + re_l^{r-1} e'_l]}{[e_h^r + e_l^r]^3} \\ &\geq (1-p_l) \frac{re_l^{r-1} re_h^{r-1} e'_h [e_h^r + e_l^r] - 2re_l^{r-1} re_h^{2r-1} e'_h}{[e_h^r + e_l^r]^3} \\ &= (1-p_l) re_l^{r-1} re_h^{r-1} e'_h \frac{e_l^r - e_h^r}{[e_h^r + e_l^r]^3} \geq 0, \end{aligned}$$

where the last inequality holds true by (25) and $e'_h \leq 0$. Therefore, $e'_l \leq 0$ and $e'_h \leq 0$ lead to a contradiction and the result follows. ■

Using (22), (23) and Lemma 5, we are finally ready to proceed with the four main steps needed to prove Theorem 2 and Proposition 3. The proofs of the four steps are alike. First, we simplify the difference in the expected sum of efforts under the two disclosure policies using (6), (22) and (23). Second, we use Lemma 5 to conclude the proofs.

Step 1. $\pi^{\{\mathcal{D}, \mathcal{D}, \mathcal{D}\}} > \pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}}$.

In this step we denote by e_h and e_l the efforts under \mathcal{C} and policy $\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$; that is, it is commonly known that the high-type does not know her rival's type and the low-type knows her rival's type. The claim is equivalent to

$$\begin{aligned} p^2(2e_{hh} - 2e_h) + 2p(1-p)(e_{hl} + e_{lh} - e_h - e_l) &> 0 \\ pe_{hh} + (1-p)(e_{hl} + e_{lh}) - (1-p)e_l - e_h &> 0 \\ p \frac{rh}{4} V + (1-p)r \frac{h^r l^r (h+l)}{(h^r + l^r)^2} V - (1-p)e_l - e_h &> 0, \end{aligned}$$

¹⁹Recall that we are under assumptions $e'_l \leq 0$ and $e'_h \leq 0$.

where we used (3) in the last step. Now, use (22) to eliminate e_h , and obtain

$$\begin{aligned}
p\frac{rh}{4}V + (1-p)r\frac{h^r l^r (h+l)}{(h^r + l^r)^2}V - (1-p)e_l - (1-p)\frac{h}{l}e_l - p\frac{rh}{4}V &> 0 \\
r\frac{h^r l^r (h+l)}{(h^r + l^r)^2}V - \frac{h+l}{l}e_l &> 0 \\
r\frac{h^r l^{r+1}}{(h^r + l^r)^2}V &> e_l \\
e_{lh} &> e_l, \quad (31)
\end{aligned}$$

where the last step is implied by (3).

Now, the low-type is sure she is up against a high-type in both e_{lh} and e_l , but in the former the high-type's belief of being up against another high-type is 0 while in the latter it is p . Therefore, (31) follows from the first statement of the Lemma 5.

Step 2. $\pi^{\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}} > \pi^{\{\mathcal{D}, \mathcal{D}, \mathcal{D}\}}$.

In this step we denote by e_h and e_l the efforts under \mathcal{C} and policy $\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$; that is, it is commonly known that the high-type knows her rival's type and the low-type does not know her rival's type. The claim is equivalent to

$$\begin{aligned}
2p(1-p)(e_h + e_l - e_{hl} - e_{lh}) + (1-p)^2(2e_l - 2e_{ll}) &> 0 \\
e_l + pe_h - p(e_{hl} + e_{lh}) - (1-p)e_{ll} &> 0 \\
e_l + pe_h - pr\frac{h^r l^r (h+l)}{(h^r + l^r)^2}V - (1-p)\frac{rl}{4}V &> 0,
\end{aligned}$$

where we used (3) in the last step. Now, use (23) to eliminate e_l , and obtain

$$\begin{aligned}
p\frac{l}{h}e_h + (1-p)\frac{rl}{4}V + pe_h - pr\frac{h^r l^r (h+l)}{(h^r + l^r)^2}V - (1-p)\frac{rl}{4}V &> 0 \\
e_h &> r\frac{h^{r+1}l^r}{(h^r + l^r)^2}V \\
e_h &> e_{hl}, \quad (32)
\end{aligned}$$

where the last step is implied by (3).

Now, the high-type is sure she is up against a low-type in both e_h and e_{hl} , but in the former the low-type's belief of being up against another low-type is $1-p$ while in the latter it is 0. Therefore, (32) follows from the second statement of the Lemma 5.

Step 3. $\pi^{\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}} > \pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}}$.

Now both disclosure policies include some concealment, and hence there are two efforts under \mathcal{C} for each type according to the disclosure policy. In this step, we denote by \hat{e}_h and \hat{e}_l the efforts under \mathcal{C} and policy $\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$, that is when it is commonly known that the high-type does not know her rival's type and the low-type knows her rival's type ($p_h = 0$ and $p_l = 1-p$), and by \bar{e}_h and \bar{e}_l the efforts

under \mathcal{C} and policy $\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}$, that is when it is commonly known that no type knows her rival's type ($p_h = p$ and $p_l = 1 - p$). The claim is equivalent to

$$\begin{aligned} p^2(2e_{hh} - 2\bar{e}_h) + 2p(1-p)(\hat{e}_h + \hat{e}_l - \bar{e}_h - \bar{e}_l) + (1-p)^2(2\hat{e}_l - 2\bar{e}_l) &> 0 \\ p^2\frac{rh}{4}V + p(1-p)\hat{e}_h - p\bar{e}_h + (1-p)\hat{e}_l - (1-p)\bar{e}_l &> 0, \end{aligned}$$

where we used (3). Now, use (23) for policy $\{\mathcal{D}, \mathcal{C}, \mathcal{C}\}$ to eliminate \hat{e}_h , and (6) for policy $\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}$ to eliminate \bar{e}_h , and obtain

$$\begin{aligned} p^2\frac{rh}{4}V + (1-p)\frac{h}{l}\hat{e}_l - (1-p)^2\frac{rh}{4}V - (1-p)\frac{h}{l}\bar{e}_l + (1-2p)\frac{rh}{4}V + (1-p)\hat{e}_l - (1-p)\bar{e}_l &> 0 \\ (1-p)\frac{h}{l}\hat{e}_l - (1-p)\frac{h}{l}\bar{e}_l + (1-p)\hat{e}_l - (1-p)\bar{e}_l &> 0 \\ \hat{e}_l &> \bar{e}_l, \end{aligned}$$

Now, the low-type knows she is up against a high-type with probability p in both \hat{e}_l and \bar{e}_l , but in the former the high-type's belief of being up against another high-type is 0 while in the latter it is p . Therefore, $\hat{e}_l > \bar{e}_l$ follows from the first statement of the Lemma 5.

Step 4. $\pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}} > \pi^{\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}}$.

Now both disclosure policies include some concealment, and hence there are two efforts under \mathcal{C} for each type according to the disclosure policy. In this step we denote by \hat{e}_h and \hat{e}_l the efforts under \mathcal{C} and policy $\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$, that is when it is commonly known that the high-type does not know her rival's type and the low-type knows her rival's type ($p_h = p$ and $p_l = 0$), and by \bar{e}_h and \bar{e}_l the efforts under \mathcal{C} and policy $\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}$, that is when it is commonly known that no type knows her rival's type ($p_h = p$ and $p_l = 1 - p$). The claim is equivalent to

$$\begin{aligned} p^2(2\hat{e}_h - 2\bar{e}_h) + 2p(1-p)(\hat{e}_h + \hat{e}_l - \bar{e}_h - \bar{e}_l) + (1-p)^2(2e_{ll} - 2\bar{e}_l) &< 0 \\ (1-p)^2\frac{rl}{4}V + p\hat{e}_h - p\bar{e}_h + p(1-p)\hat{e}_l - (1-p)\bar{e}_l &< 0, \end{aligned}$$

where we used (3). Now, use (22) for policy $\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$ to eliminate \hat{e}_l , and (6) for policy $\{\mathcal{C}, \mathcal{C}, \mathcal{C}\}$ to eliminate \bar{e}_l , and obtain

$$\begin{aligned} (1-p)^2\frac{rl}{4}V + p\hat{e}_h - p\bar{e}_h + p\frac{l}{h}\hat{e}_h - p^2\frac{rl}{4}V - p\frac{l}{h}\bar{e}_h - (1-2p)\frac{rl}{4}V &< 0 \\ p\hat{e}_h - p\bar{e}_h + p\frac{l}{h}\hat{e}_h - p\frac{l}{h}\bar{e}_h &< 0 \\ \bar{e}_h &> \hat{e}_h. \quad (33) \end{aligned}$$

Now, the high-type knows she is up against another high-type with probability p in both in both \bar{e}_h and \hat{e}_h , but in the former the low-type's belief of being up against another low-type is 0 while in the latter it is $1 - p$. Therefore, (33) follows from the second statement of the Lemma 5.

Corner Equilibria. Finally, we are left to show that considering corner equilibria does not affect the main results. In any corner equilibria, it cannot be that

$e_h = 0$, as it would contradict $e_h > e_l$ (see (25) in the proof of Lemma 5). When $e_l = 0$, the ranking of policies of Proposition 3, and therefore Theorem 2, are not affected. It could be proved that $e_l = 0$ can be sustained in equilibrium only if $r = 1$, the belief of the high-type of being up against another high-type is greater than $4l/h$, and only when $\mathcal{P} = \{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$, so that the low-type infers to be up against a high-type and the high-type believes she is up against a high-type with probability p . The reason why corner equilibria do not affect the results can be immediately noticed in the only two steps where disclosure policy $\{\mathcal{C}, \mathcal{C}, \mathcal{D}\}$ plays a role; the proof of Step 1 in the proof of Theorem 2, which carries over to the case of $e_l = 0$, and the proof of Step 4 in the proof of Theorem 2, which carries over to the case of $\hat{e}_l = 0$.

Appendix B. Extensions.

In this Appendix, we test the robustness of Theorem 1 along five dimensions.

(B.1) Ex-post Disclosure. We assumed the principal has ex-ante commitment power. If the principal lacks such commitment power, then she might have an incentive to renege on her ex-ante announced disclosure policy once she observes types. Hence, the disclosure policy is effectively chosen *ex-post*. In what follows, we claim that concealment under $\{h, h\}$ or $\{l, l\}$ is never profitable for the principal, hence concealment in the only remaining contingency $\{h, l\}$ makes contestants perfectly infer types, and thus contestants always exert their equilibrium efforts under full disclosure.

First, when the principal observes $\{h, h\}$, she is strictly better-off fully disclosing types regardless of what she announced before observing types and regardless of the contestants' beliefs if she had instead concealed types; in fact, the maximum effort for a high-type is exerted when she is certain she is up against another high-type under disclosure. Formally, e_{hh} is the highest effort a high-type ever exerts (see the expression of e_{hh} in (3) and (29) in the proof of Lemma 5). Thus, the principal is strict better-off disclosing upon observing $\{h, h\}$ unless $p = 1$, in which case any disclosure policy reaches the maximum e_{hh} .

Analogously, when the principal observes $\{l, l\}$, she is also strictly better-off fully disclosing types; in fact, the maximum effort for a low-type is exerted when she is certain she is up against another low-type under disclosure (see the expression of e_{ll} in (3) and (29) in the proof of Lemma 5). This is true unless $p = 0$, in which case any disclosure policy reaches the maximum e_{ll} .

Therefore, ruling out the trivial $p \in \{0, 1\}$ cases, the principal might be weakly better-off concealing types only when types are $\{h, l\}$. However, the ex-post concealment conveys itself information to contestants, because they know that the principal was aware of the realization of types when she chose to conceal. Since $\{h, l\}$ is the only contingency where the principal might conceal, contestants infer that concealment implies that types are $\{h, l\}$. Therefore, under ex-post disclosure contestants' types are always common-knowledge, and thus contestants exert \mathcal{D} -equilibrium efforts.

Finally, since with ex-ante commitment power the principal can always achieve the \mathcal{D} -equilibrium efforts, if the principal could choose to commit ex-ante to a disclosure policy, or wait to first observe types and then choose a disclosure policy, she never has an incentive to wait and observe types.

(B.2) Principal's Objective Function. We assumed the principal maximizes aggregate effort. There are plausible alternatives. First, if one has in mind harmful conflicts or wasteful lobbying, the principal minimizes aggregate effort; the ranking of policies we retrieved would then be simply inverted. Second, if one has in mind contests for hiring new employees or the economics job market, the principal aims for selection of the most skilled contestant as the winner; when the principal maximizes the probability of a high-type winner, the optimal disclosure policy is \mathcal{D} if $p \in (0, 0.5)$, \mathcal{C} if $p \in (0.5, 1)$, and indifference if $p \in \{0, 0.5, 1\}$.²⁰ Third, in an

²⁰An easy way to see this is as follows. The principal's disclosure policy affects the probability

architectural or research contest the principal will eventually only implement the winning project; when the principal maximizes the expected winning effort (EWE), she favours heterogeneity of efforts more than when maximizing aggregate effort (see Serena (2017) for the intuition), and situations of heterogeneity of efforts arise only in uneven contests — $\{h, l\}$ or $\{l, h\}$. Since in uneven contests the principal is better-off having committed to conceal types, EWE-maximization increases the incentive to conceal types as opposed to the standard aggregate effort maximization. In fact, the optimal disclosure policy becomes to commit to full concealment (disclosure) if $p \in [0, \bar{p}] \cup \{1\}$ ($p \in [\bar{p}, 1]$), where $\exists! \bar{p} \in (\frac{1}{2}, 1]$.

(B.3) Continuum of Types. We assumed a binary type space and found a tight link between the relative likelihood of high- and low-types and the optimal disclosure policy (see Theorem 1). A natural question is whether this link carries over with a continuum of types. Due to the well-known lack of tractability of private information contests with a continuum of types, we rely on two specific results from the literature. First, Ewerhart (2010) finds that under a particular continuous distribution of types, the Tullock contest with $r = 1$ does admit a closed-form solution for equilibrium efforts. His results can be used to show that the link between the relative likelihood of types and optimal disclosure policy that we found goes in the same direction of his special case of continuum type space. In particular, in Ewerhart’s distribution lower types more likely than higher types, and in fact numerical simulations, available upon request, on Ewerhart’s distribution show that the aggregate effort is greater under concealment than under disclosure, thus going in the same direction of the finding of our Theorem 1. Second, Wasser (2013), in his Table 1, provides numerical simulations of aggregate effort under complete and private information when the prior follows a Beta distribution. Wasser’s numerical simulations show that if the skewness of the Beta distribution is high (low), such that the relative likelihood of higher types over that of lower types is high (low), then the aggregate effort under complete information is greater (smaller) than under private information, thus going in the same direction of the finding of our Theorem 1.

(B.4) General Contest Success Function. We assumed $p_i(e_i, e_j) = e_i^r / (e_i^r + e_j^r)$ with $r \leq 1$. A generalization to $p_i(e_i, e_j) = f(e_i) / (f(e_i) + f(e_j))$, under some carefully chosen properties of the $f(\cdot)$ function is difficult to achieve. The reason is as follows. Indifference between disclosure and concealment is achieved when *concealment- e_h* and *disclosure- e_h* coincide (see Figure 1), and the same condition holds for a low-type (see Figure 2). Where and how many times do *concealment- e_h* and *disclosure- e_h* cross? Thanks to assumption that $f(e_i) = e_i^r$ with $r \leq 1$,

(A) *concealment- e_h* is concave in p , and thus there is a unique $p \in (0, 1)$ for which it equals *disclosure- e_h* , which is a straight line.

(B) *concealment- e_l* is convex in p , and thus there is a unique $p \in (0, 1)$ for which it equals *disclosure- e_l* , which is a straight line.

of a high-type winner only when the contest is uneven, which occurs with probability $2p(1 - p)$. Thus, besides the usual indifference when $p \in \{0, 1\}$, full concealment dominates full disclosure if and only if $\frac{e_h}{e_h + e_l} \geq \frac{e_{hl}}{e_{hl} + e_{lh}} = \frac{h}{h+l}$, or equivalently $\frac{e_h}{e_l} \geq \frac{h}{l}$. A straightforward consequence of (6) is that $\frac{e_h}{e_l} \geq \frac{h}{l} \iff p \geq \frac{1}{2}$, and thus the result follows.

(C) these two unique values of p both occur in $p = 0.5$.²¹
 More general standard regularity conditions on the function $f(\cdot)$, such as

$$(Reg): f(0) = 0, f'(\cdot) > 0 \text{ and } f''(\cdot) \leq 0,$$

guarantee none of (A)–(B)–(C), and thus it undermines the fact that $\pi^{\mathcal{D}-\mathcal{C}} = 0$ in a unique point (i.e., in $p = 0.5$). Can we find an assumption milder than $f(e_i) = e_i^r$ and stronger than (Reg) which would still make $\pi^{\mathcal{D}-\mathcal{C}} = 0$ for a unique p — perhaps different than 0.5? Guaranteeing (A)–(B) could be easily done with a mild condition on f''' .²² Nevertheless, without (C), uniqueness of p for which $\pi^{\mathcal{D}-\mathcal{C}} = 0$ is not granted, and $\pi^{\mathcal{D}-\mathcal{C}}$ might cross the horizontal axis several times. In fact, if say $p = 0.3$ satisfies $concealment - e_h = disclosure - e_h$ and $p = 0.7$ satisfies $concealment - e_l = disclosure - e_l$, then there could be multiple $p \in [0.3, 0.7]$ such that $\pi^{\mathcal{D}-\mathcal{C}}$ changes sign.

²¹In fact, $p = 0.5$ yields $\pi^{\mathcal{D}-\mathcal{C}} = 0$. This can be immediately seen in Proposition 5 of Malueg and Yates (2004).

²²In order to sign $\frac{\partial^2 e_h(p)}{\partial p^2}$ one could differentiate (4) and (5) with respect to p twice. It is clear that it would be an expression in f' , f'' and f''' . Conditions on f' and f'' are already imposed by (Reg). Appropriate conditions on f''' would drive the sign of $\frac{\partial^2 e_h(p)}{\partial p^2}$.

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